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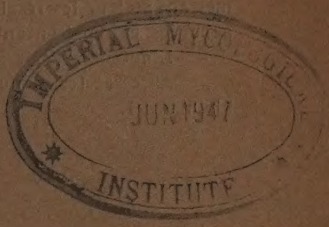

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MATHEMATICS

A NEW PROOF OF THE THEOREM ABOUT THE THREE GEODESICS

By L. LUSTERNIK

(Communicated by S. L. Soboleff, Member of the Academy, 6. IV. 1943)

In a paper (1) by L. Schnirelman and the author the following theorem was proved.

For every thrice differentiable surface S of genus O one of the following mutually exclusive cases takes place:

1) there exist three non-self-intersecting closed geodesics of different lengths;

2) there exists a one-parameter family of such geodesics of equal length covering the surface and one such geodesic of different length;

3) all the geodesics are closed non-self-intersecting curves of equal length.

The cases 1, 2 and 3 are realized, for instance, on an ellipsoid with three axes, on an ellipsoid of revolution (or, generally, on an arbitrary surface of revolution) and on a sphere, respectively.

This theorem was proved with the use of the homotopical invariant «category». This theorem can also be obtained, without changing the analytical part of the proof in the above paper (1), as a particular case of a general theorem in our previous note. The latter establishes the connexion between the number of solutions of a variational problem and the ring of intersections in the functional space of extremals of this problem.

The surface S may be represented as a sphere with the given Riemann metric. We shall denote by z_3 the set of all circles on the sphere; by z_2^a, the set of all circles on S passing through a fixed point a of the sphere; by z_1^{ab}, the set of all circles, passing through the points a and b of the sphere.

Functional Space P_S. For any pair of closed curves q and q_1 on S the distance $\rho(q, q_1)$ can be defined. Namely, we introduce some parameters on q and q_1 and the cyclic coordinates φ and φ_1 of the same period, then we take the maximum of distances from the points of q to those of q_1 with the same coordinates and define $\rho(q, q_1)$ as the greatest lower bound of all such maxima corresponding to the various parametrizations of q and q_1. Let P_S be the totality of rectifiable curves on S. Introducing into P_S the above notion of distance, we convert P_S into a metric space. If P_S^N denotes the set of curves from P_S, the lengths of which do not exceed N, then P_S^N is a compactum. If N is sufficiently large, then P_S^N contains the set z^3 and, consequently, all z_2^a and z_1^{ab}. Consider the homologies modulo 2 in P_S^N. z_2^0, z_3^a and z_1^{ab} are Δ-cycles in P_S^N of dimensions 1, 2 and 3, respectively. z_1^{ab} is included in z_3^a which is included in z_2^0 in the set-theoretical sense. We shall prove the analogous re-

lations of «subordination» (2) between these cycles, i. e. the existence of a one-dimensional ∇-cycle Y_1^a in P_S^N such that

$$Y_3^a \times z_3 = z_2^a.$$

Upper Cycle Y_1^a. Let a be an arbitrary point of S; q and q_1 — any curves from P_S^N, the distance of which is less than a given positive number ε. If ε is sufficiently small, then q and q_1 can be covered by a «film» (the latter can be obtained, for instance, as follows: introduce the cyclic coordinates on q and q_1 such that the distances between the points of the curves having the same coordinates do not exceed ε, and take the totality of geodesics joining the points of q and q_1 with equal coordinates). If both q and q_1 do not pass through the point a, then the degree of covering $f^\varepsilon(q, q_1)$ of a by this film, which is either 0 or 1, does not depend on the choice of parameters on q and q_1, whenever the distances between the corresponding points of the curves do not exceed ε. The sequence of functions f_N^ε at $\varepsilon \to 0$ determines a one-dimensional ∇-cycle in P_S that we denote by Y_1^a. As $\varkappa(Y_1^a, z_1^{bc}) = 1$, that can be easily verified, both Y_1^a and z_1^{bc} are not homologous to zero in P_S^N. We further see that $Y_1^a \times z_3 = z_2^a$; $Y_1^b \times z_2^a = z_1^{ab}$; $(Y_1^a \times Y_1^b) \times z_3 = z_1^{ab}$, i. e. z_2^a (resp. z_1^{ab}) is subordinate to z_3 (resp. to z_2^a) by one-dimensional ∇-cycles of homology class (Y_1^a), and the cycle z_1^{ab} is subordinate to z_3 by two-dimensional ∇-cycles $(Y_1^a \times Y_1^b)$.

The set $M \subset P_S^N$ will be said to be a K-set, if it possesses the properties:

1) M consists of non-self-intersecting curves;

2) M can be obtained from a closed subset of z_3 by means of a continuous deformation in P_S^N;

3) the length $l(q_1)$ of an arc q' of a curve q from M is a functional of the arc q' when the latter ranges through all possible arcs of the curves from M.

According to the definition of the K-critical set given in (2), the K-critical set is the set of closed non-self-intersecting geodesics of equal lengths. This can be understood as follows.

For the given c and any $\varepsilon > 0$ there exists $\alpha > 0$ such that if the distance from every curve of length $\geqslant c - \alpha$ belonging to a K-set A to a closed geodesic is greater than ε, then the family of curves A can be deformed into a family of curves A', which is also a K-set and consists of the curves of length $\leqslant c - \alpha_1 < c$ (this means that the longest curves of the family A, being sufficiently «non-geodesical», can be shortened enough by means of a continuous deformation, so that they remain without self-intersections). This theorem is proved in the paper (1) mentioned above, where the required deformation is constructed.

Let now $[z_3]$, $[z_2]$, $[z_1]$ be the K classes of Δ-cycles in P_S^N, where the cycles of classes $[z_i]$ $(i = 1, 2, 3, \ldots)$ are homologous to z_3, z_2^a, z_1^{ab}, respectively. These classes are subordinate to one another by means of the cycles Y^a, $(Y_1^a \times Y_1^b)$. If c_1, c_2, c_3 denote the greatest lower bounds of the maxima of lengths of the curves on the cycles of classes $[z_1]$, $[z_2]$, $[z_3]$, then $c_1 \leqslant c_2 \leqslant c_3$.

Three cases are possible:

1) $c_1 < c_2 < c_3$;

2) $c_1 = c_2 < c_3$ or $c_1 < c_2 = c_3$;

3) $c_1 = c_2 = c_3$.

In virtue of the fundamental theorem in (2) the three cases of the theorem formulated above correspond to these three cases.

Stekloff Mathematical Institute.
Academy of Sciences of the USSR.

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MATHÉMATIQUES

THÉORÈME D'EXISTENCE D'UN COUPLE DE CONGRUENCES NON-ORTHOGONAL, BILATÉRALEMENT STRATIFIABLE DONT LA CONGRUENCE DES PERPENDICULAIRES COMMUNES EST ISOTROPE

Par S. ROSSINSKI

(Présenté par A. N. Kolmogoroff, de l'Académie, le 15. XII. 1942)

§ 1. Pendant presqu'une vingtaine d'années écoulées depuis l'apparition en 1924 des deux mémoires de L. Bianchi (1) publiés dans les Atti dei Lincei et qui ont été l'origine d'une nouvelle voie dans la théorie des congruences, la théorie métrique des couples de congruences stratifiables a été l'objet d'une étude détaillée des géomètres soviétiques et étrangers.

Dans l'URSS le premier travail consacré à la théorie métrique des couples stratifiables (2), publié en 1929, appartient à l'auteur du présent article; il résout le problème de la recherche des couples stratifiables dont la congruence des perpendiculaires communes est n o r m a l e. Ce problème contient comme le cas particulier le plus simple—celui qui a été étudié dans le premier mémoire de Bianchi.

§ 2. Dans le présent article l'auteur pose le problème suivant—démontrer l'existence d'un couple stratifiable de congruences dont la congruence des perpendiculaires communes est isotrope.

La démonstration de l'existence d'un objet géométrique peut être réduite par la méthode contemporaine de M. Cartan à l'analyse d'un certain système d'équations de Pfaff. Il y a des cas où la structure du système des équations quadratiques extérieures pour le système de Pfaff donné est telle qu'on peut immédiatement en conclure que l'objet géométrique considéré existe et trouver le degré de généralité qu'on a en déterminant cet objet par le système de Pfaff donné. Dans ces cas les difficultés qu'un savant qui s'occupe de géométrie différentielle rencontre dans le domaine de l'analyse disparaissent, mais ces cas sont très rares, et en particulier dans les problèmes concrets de la géométrie.

Or, les cas où le système donné de Pfaff n'est pas en involution peuvent conduire à des difficultés d'ordre analytique qu'il est souvent très difficile à surmonter. C'est pourquoi les cas où l'étude d'un tel système de Pfaff conduit à un résultat déterminé sont peu nombreux dans l'état actuel de la science. Dans le problème de la démonstration de l'existence d'un objet géométrique, basée sur la considération d'un système de Pfaff qui n'est pas en involution, on ne connaît que les résultats de M. Cartan lui-même qui a créé cette méthode—son mémoire

célèbre sur la déformation projective des surfaces publié en 1920 (3)—et les recherches de M. Pantazi apparues dans la dernière dizaine d'années concernant la théorie projective des couples stratifiables et où la méthode de Cartan est systématiquement appliquée.

Dans la littérature mathématique soviétique il nous paraît qu'il n'a été publié aucun article où l'existence d'un objet géométrique est démontrée en prenant pour base l'étude d'un système de Pfaff qui n'est pas en involution et qui, enrichi par des équations supplémentaires que l'on obtient en appliquant plusieurs fois le lemme de Cartan, se réduit à un système complètement intégrable.

§ 3. Soit φ un angle d'un trièdre biorthogonal, en général il n'est pas un angle droit; désignons par M le sommet du trièdre; par i_1 et i_2 les vecteurs unitaires dirigés le long de ces arêtes, qui ne sont pas mutuellement orthogonales; par i_3 le vecteur unitaire dirigé le long de la troisième arête.

Convenons d'établir une correspondance entre une famille de trièdres et les rayons d'une congruence donnée, en exigeant seulement que la troisième arête du trièdre biorthogonal coïncide avec un rayon de la congruence.

En désignant par la même lettre M le vecteur du point M, écrivons les formules qui définissent les déplacements infiniment petits du trièdre:

$$dM = \omega^\alpha i_\alpha, \quad di_k = \omega_k^\alpha i_\alpha, \qquad (k = 1, 2, 3)$$

où α est l'indice de sommation.

En posant

$$i_1 i_2 = \cos\varphi = p,$$

nous trouvons les conditions

$$\left.\begin{aligned} \omega_1^1 &= -p\omega_1^2 \\ \omega_2^2 &= -p\omega_2^1 \\ \omega_3^3 &= 0 \end{aligned}\right\} \qquad (1)$$

$$\omega_3^1 = \frac{p\omega_2^3 - \omega_1^3}{1-p^2}, \quad \omega_3^2 = \frac{p\omega_1^3 - \omega_2^3}{1-p^2}, \qquad (2)$$

$$dp = (1-p^2)(\omega_1^2 + \omega_2^1), \qquad (3)$$

qui donnent les relations entre les formes ω.

Dans la suite la congruence liée de la manière indiquée avec le repère biorthogonal sera nommée congruence (K_3).

§ 4. Soit (K_3) la congruence des perpendiculaires communes des rayons correspondants d'un certain couple de congruences (K_1) et (K_2) bilatéralement stratifiable, et soit P_i le point d'intersection d'un rayon K_i avec le rayon K_3 qui est perpendiculaire à ce rayon.

Plaçons le sommet M du repère biorthogonal dans le point qui est le milieu du segment P_1P_2 et donnons aux arêtes i_1 et i_2 des directions respectivement parallèles aux rayons des congruences K_1 et K_2 du couple considéré. Désignons ensuite les vecteurs

$$\overrightarrow{MP_1} = ai_3, \quad \overrightarrow{MP_2} = -ai_3.$$

Quand (K_1), (K_2) forment un couple bilatéralement stratifiable dont (K_3) est la congruence des perpendiculaires communes, les formes ω, da et dp sont déterminées par le système de Pfaff formé des équations (1), (2) et (3) et des suivantes:

$$\begin{aligned} \omega^3 - da &= A\omega_1^3 + B\omega_2^3, & -2a\omega_2^1 &= x\omega_1^3 + y\omega_2^3, \\ +2a\omega_1^2 &= B\omega_1^3 + C\omega_2^3, & \omega^3 + da &= y\omega_1 + z\omega_2^3, \\ \omega^1 &= u\omega_1^3 - v\omega_2^3, & \omega^2 &= v\omega_1^3 - w\omega_2^3, \end{aligned}$$

$$(xw - zu)(1-p^2) + a(x-z) = 0, \qquad Cx - Az = 0.$$
$$(Aw - Cu)(1-p^2) + a(A-C) = 0,$$

Ce système dans la forme que nous venons d'indiquer a été considéré en 1939 dans un article de M. Bachvaloff [4], mais M. Pantazi [5] avait déjà publié en 1932 un système de Pfaff analogue pour la théorie générale des couples stratifiables, théorie basée sur l'application d'un repère à trois paramètres.

§ 5. L'abscisse r du centre d'un rayon de (K_3) est déterminée par l'égalité

$$[r - u(1-p^2)]\,\omega_1^3\omega_1^3 - 2pr\omega_1^3\omega_2^3 + [r + w(1-p^2)]\,\omega_2^3\omega_2^3 = 0.$$

Si (K_3) est une congruence isotrope, on a

$$r - u(1-p^2) = 0, \quad pr = 0, \quad r + w(1-p^2) = 0,$$

ce qui donne deux cas essentiellement différents.

1°. Couple orthogonal *. On a ici

$$p = 0, \quad r = u = -w,$$

c'est-à-dire *les rayons correspondants du couple stratifiable doivent être mutuellement perpendiculaires et le milieu du rayon de la congruence isotrope* (K_3) *ne doit pas, en général, coïncider avec le milieu du segment* P_1P_2.

2°. Couple non-orthogonal. Dans ce cas $r = 0$ et comme on a évidemment $p \neq \pm 1$, il en résulte $u = w = 0$, c'est-à-dire *un couple de congruences stratifiable non-orthogonal dont la congruence des perpendiculaires communes est isotrope jouit de la propriété: le milieu du segment* P_1P_2 *de la perpendiculaire commune des rayons correspondants du couple décrit la surface moyenne de la congruence isotrope.*

§ 6. Le système de Pfaff qui définit le couple non-orthogonal stratifiable cherché est formé des équations (1), (2) et des suivantes:

$$\left.\begin{aligned} dp &= \frac{1-p^2}{2a}\{(B-x)\,\omega_1^3 + (A-y)\,\omega_2^3\}, \\ \omega^3 - da &= A\omega_1^3 + B\omega_2^3, \qquad -2a\omega_2^1 = x\omega_1^3 + y\omega_2^3, \\ 2a\omega_1^2 &= B\omega_1^3 + A\omega_2^3; \qquad \omega^3 + da = y\omega_1^3 + x\omega_2^3, \\ \omega^1 &= -\varphi\omega_2^3, \quad \omega^2 = \varphi\omega_1^3 \end{aligned}\right\} \qquad (4)$$

et il n'est pas en involution.

Pour trouver le degré de généralité qu'on a en déterminant les couples étudiés, il faut enrichir le système fondamental des équations (1), (2) et (4) par des équations obtenues en appliquant le lemme de M. Cartan.

Cela nous donne 4 équations nouvelles de Pfaff pour dx, dy, dA, dB avec 4 nouveaux paramètres α, β, λ, μ, liés par une certaine relation algébrique avec x, y, A, B, a, p.

Les équations quadratiques extérieures de ces quatre conduisent à 4 équations de Pfaff pour $d\alpha$, $d\beta$, $d\lambda$, $d\mu$ avec 4 nouveaux paramètres α_1, β_1, λ_1, μ_1, liés par deux relations algébriques avec α, β, λ, μ, x, y, A, B, a, p, φ.

L'enrichissement ultérieur du système conduit à 4 équations de Pfaff pour $d\alpha_1$, $d\beta_1$, $d\lambda_1$, $d\mu_1$ avec des nouveaux paramètres α_{11}, β_{11}, λ_{11}, μ_{11}, liés par quatre relations algébriques avec α_1, β_1, λ_1, μ_1, α, β, λ, μ, x, y, A, B, a, p, φ.

Les 4 dernières relations sont de telle nature, que l'on peut en exclure α_{11}, β_{11}, λ_{11}, μ_{11}, qui n'entrent que sous la forme de sommes $\alpha_{11} + \beta_{11}$ et $\lambda_{11} + \mu_{11}$

* Dans l'article de M. Finikoff [6] l'existence des couples orthogonaux est démontrée; cependant les résultats obtenus tombent en défaut pour les couples ayant une congruence isotrope de perpendiculaires communes, parce que le trièdre de Guichard introduit par l'auteur ne peut pas être appliqué dans le cas d'une congruence isotrope. Dans mon article [7] j'ai proposé un repère triorthogonal qui peut, à mon avis, être appliqué à toutes les congruences non dégénérées et peut probablement être utilisé avec succès dans l'analyse de plusieurs problèmes de la théorie des congruences.

et donnent deux relations algébriques nouvelles entre les paramètres α_1, β_1, λ_1, μ_1, α, β, λ, μ, x, y, A, B, a, p, φ.

Cependant les quatre relations algébriques entre α_1, β_1, λ_1, μ_1 que l'on obtient ainsi sont telles que le système algébrique linéaire par rapport aux paramètres α_1, β_1, λ_1, μ_1 ne définit pas ces derniers.

C'est pourquoi, en dérivant les relations algébriques obtenues, quand on exclut les sommes $\alpha_{11}+\beta_{11}$ et $\lambda_{11}+\mu_{11}$, on peut en vertu des équations de Pfaff et de ce que les formes ω_1^3 et ω_2^3 sont linéairement indépendantes, joindre aux quatre relations algébriques entre α_{11}, β_{11}, λ_{11}, μ_{11} quatre relations nouvelles entre ces mêmes paramètres.

En déterminant de quatre équations quelconques (dont le déterminant par rapport aux paramètres α_{11}, β_{11}, λ_{11}, μ_{11} est non nul) les valeurs de ces paramètres et en portant ces valeurs dans les équations qui restent, nous imposerons encore 4 relations aux paramètres α_1, β_1, λ_1, μ_1; elles peuvent être des conséquences des 4 premières relations ou bien les enrichir par des équations nouvelles.

Supposons d'abord que nous ne trouvons pas ainsi d'équations nouvelles pour α_1, β_1, λ_1, μ_1. Dans ce cas en portant les valeurs des paramètres α_{11}, β_{11}, λ_{11}, μ_{11} dans les équations pour $d\alpha_1$, $d\beta_1$, $d\lambda_1$, $d\mu_1$, nous arriverons à un système de Pfaff où les différentielles des inconnues a, p, φ, A, B, x, y, α, β, λ, μ, α_1, β_1, λ_1, μ_1 sont des fonctions linéaires par rapport à ω_1^3 et ω_2^3 et les coefficients *n'introduisent aucune nouvelle inconnue*. Si ce système est complètement intégrable — ce que l'on ne peut pas affirmer *a priori*, — la solution correspondante ne dépendra que de constantes arbitraires.

Supposons en second lieu qu'aux équations que l'on avait déjà et qui contiennent α_1, β_1, λ_1, μ_1 sont jointes des équations nouvelles pour les mêmes paramètres, et que du système ainsi enrichi on peut déterminer α_1, β_1, λ_1, μ_1. Cela conduira à ce que du système de Pfaff précédemment indiqué les équations pour $d\alpha_1$, $d\beta_1$, $d\lambda_1$, $d\mu_1$ disparaîtront, mais celles qui contiennent $d\alpha$, $d\beta$, $d\lambda$, $d\mu$ seront peut être conservées, si le nombre ou la forme des relations algébriques pour les paramètres α, β; λ, μ ne sont pas suffisants pour les déterminer complètement.

Donc, en tout cas nous arrivons à la conclusion, que *si la solution du problème existe, elle dépend au plus d'un certain nombre fini de constantes arbitraires.*

§ 7. Nous allons considérer maintenant les cas spéciaux. Nous nommons ainsi les cas que l'on peut obtenir du cas général (§ 6) en annulant le déterminant formé des coefficients des paramètres α_{11}, β_{11}, λ_{11}, μ_{11} dans quatre relations quelconques choisies parmi les huit relations algébriques entre α_{11}, β_{11}, λ_{11}, μ_{11}, si le déterminant par rapport à ces paramètres ne s'annule pas identiquement. Ainsi ou doit distinguer les cas spéciaux suivants:

1°. $A+x+y+B=0$;	3°. $(A+x)p+B+y=0$;	5°. $px+y=0$,
2°. $A-x+y-B=0$;	4°. $(A-x)p+B-y=0$;	6°. $pA+B=0$.

Il n'y a que trois cas essentiellement différents, à savoir 1°, 3° et 5°.

§ 8. Le cas $A+x+y+B=0$. En posant

$$A+x=-B-y=q,$$

nous introduirons dans le système (1), (2) et (4) au lieu de A et B un nouveau paramètre q.

L'étude a montré que ce cas sera caractérisé (on démontre que $q\equiv 0$) par *un système de Pfaff complètement intégrable* formé des équations (1), (2) et des suivantes:

$$
\left.
\begin{gathered}
2ad p+(1-p^2)(x+y)(\omega_1^3+\omega_2^3)=0, \\
\omega^3-da+x\omega_1^3+y\omega_2^3=0, \qquad 2a\omega_1^2+y\omega_1^3+x\omega_2^3=0, \\
-\omega^3-da+y\omega_1^3+x\omega_2^3=0, \qquad 2a\omega_2^1+x\omega_1^3+y\omega_2^3=0, \\
\omega^1+p\omega_2^3=0, \quad \omega^2-p\omega_1^3=0, \\
4ad(y-x)+\left[(1-p)(y-x)^2+(3p-1)(y^2-x^2)+\frac{8a^2}{1-p^2}\right](\omega_1^3+\omega_2^3)=0, \\
2a(y-x)d(x+y)+\left[(x+py)(y^2-x^2)-\frac{4a^2}{1-p^2}(3x+y)\right](\omega_1^3+\omega_2^3)=0.
\end{gathered}
\right\} \tag{5}
$$

Donc, le couple de congruences stratifiable cherché existe et il est déterminé avec neuf constantes arbitraires.

Il est facile de déduire des équations (5) que l'on a

$$ad p+(1-p^2)da=0,$$

d'où il résulte

$$2a=k\,\mathrm{tg}\,\frac{\varphi}{2} \qquad (k=\mathrm{const}).$$

Tel est le lien géométrique simple entre l'angle φ de deux rayons correspondants d'un couple stratifiable et leur distance minimale $2a$.

§ 9. Sur les cas $(A+x)p+B+y=0$ et $px+y=0$. Le cas $(A+x)p+B+y=0$ admet une solution particulière $A=-x$, $B=-y$, qui coïncide avec celle du § 8.

Les calculs prouvent qu'il est possible qu'il n'existe aucune autre solution effective et qu'en tout cas, si une solution plus générale que la solution particulière indiquée existe, elle ne peut dépendre que *de onze constantes arbitraires au plus.*

En ce qui concerne le cas $px+y=0$, les calculs prouvent que le problème se réduit à la fin des fins à la recherche de la compatibilité d'un grand nombre de relations algébriques très compliquées entre les paramètres. La forme de ces relations nous conduit à la conclusion probable que le cas $px+y=0$ ne donne pas de solutions effectives du problème.

Université Lomonossoff.
Moscou.

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PHYSICS

ON THE MAGNETOSTRICTION OF POLYCRYSTALS

By K. V. VLADIMIRSKIJ

(Communicated by S. I. Wawilow, Member of the Academy, 21. XII. 1942)

The equations of the theory of elasticity involving the deformations of solids in a magnetic field were dealt with by Maxwell. Further investigations established the quantitative peculiarities of the phenomenon due to the changes of state of the substance, but the shape of the general equations—the reduction of forces to tensions—remained unchanged.

In applying these equations to the interpretation of the experiment; a uniform strain produced by a uniform magnetization in an elastically uniform body is usually considered. These assumptions may be regarded as exact in some special cases only. In general; the magnetostriction represents a complicated; microscopically non-uniform deformation. The non-uniformity of the deformation is most pronounced in the case of polycrystalline materials; the mechanical forces and the elastical properties of the substance changing from point to point; according to the crystallographic orientation of the individual crystals. In the present paper the averaged values characterizing non-uniform deformations of that kind are considered.

The forces giving rise to the magnetostriction will be characterized by the tension $\tau_{ij}^{(m)}$ or by the free strains of the magnetostriction

$$e_{ij}^{(m)} = -s_{ijkl}\tau_{kl}^{(m)}; \qquad (1)$$

with which the tensions are connected through the constants of elasticity.

The body forces and surface tractions are denoted by $X_i^{(m)}$ and $\Xi_i^{(m)}$, respectively. For ferromagnetic substances in fields of order up to 10^3 oersted the tensions $\tau_{ij}^{(m)}$ may be taken to depend solely on the direction of the spontaneous magnetization at a given point. The values of $e_{ij}^{(m)}$ may then be determined from the measurements of the magnetostriction of uniformly magnetized single crystals. In larger fields the paramagnetic magnetostriction and Maxwell's tensions are to be taken into account. We shall assume for the future that the values of $e_{ij}^{(m)}$ or $\tau_{ij}^{(m)}$ are given at each point of the body.

If the strain is uniform and no other external force is present, the components of strain e_{ij} are simply equal to the corresponding components of $e_{ij}^{(m)}$. In the general case of a non-uniform deformation the values of the strain components differ essentially from the values of $e_{ij}^{(m)}$ at the same points, owing to the elastic interaction of the neighbouring domains of the body. The expe-

rimentally measured magnetostriction represents then the average value of the corresponding strain component

$$\frac{\delta l}{l} = \frac{1}{V} \int e_{11} d\varrho; \qquad (2)$$

the axis of OX_1 is defined to coincide with the direction of the measurement.

The deformation is assumed to be quasi-uniform, i. e. the result of averaging does not depend upon the choice of the volume V, provided that its dimensions are great enough as compared with that of the irregularities of strain. In particular, the integration may be extended throughout the entire volume of the body.

The calculation of the average values proceeds by means of Betti's reciprocal theorem ([1]). To find the average strains Betti introduces an auxiliary system of uniform stresses of the form $\tau'_{ij} = \delta_{1i}\delta_{1j}$. Such a state of stress is, generally speaking, impossible in the case of elastically non-uniform bodies we are interested in. We express the volume integral (2) purely kinematically in terms of surface displacements

$$\frac{\delta l}{l} = \frac{1}{V} \int \frac{\partial}{\partial x_i} (u_1 \delta_{1i})\, d\varrho = \frac{1}{V} \int u_1 n_1\, dS, \qquad (3)$$

and consider the right-hand member of (3) to be the work done by the auxiliary system of forces $\Xi'_1 = n_1$, $\Xi'_2 = \Xi'_3 = 0$ applied, as in Betti's case, at the surface of the body alone. The corresponding auxiliary system of stresses, which is non-uniform in general, will be denoted by τ'_{ij}; n_i is here the unit vector of the external normal. Using the reciprocal theorem, we may write

$$\frac{\delta l}{l} = \frac{1}{V} \int X_i^{(m)} u'_i\, d\varrho + \frac{1}{V} \int \Xi_i^{(m)} u'_i\, dS, \qquad (4)$$

or, expressing the forces in terms of tensions and using Green's theorem,

$$\frac{\delta l}{l} = \frac{1}{V} \int \frac{\partial \tau_{ij}^{(m)}}{\partial x_j} u'_i\, d\varrho - \frac{1}{V} \int \tau_{ij}^{(m)} n_j u'_i\, dS = -\frac{1}{V} \int \tau_{ij}^{(m)} e'_{ij}\, d\varrho. \qquad (5)$$

Making use of the relation (1), we find

$$\frac{\delta l}{l} = \frac{1}{V} \int e_{ij}^{(m)} \tau'_{ij}\, d\varrho. \qquad (6)$$

We may express the result by saying, that *the macroscopical magnetostriction of polycrystals is expressed in terms of the free strains of the magnetostriction* $e_{ij}^{(m)}$ *as the volume average with the weighting factor* τ'_{ij}. *The physical meaning of this factor is that of the stress arising at a given point of the body when extended by the forces applied at the external surface.*

In the papers dealing with magnetostriction it has been assumed; since Akulov's paper ([2]) appeared, that the magnetostriction answering to any spatial distribution of the magnetization is equal to the volume average of $e_{11}^{(m)}$. Equation (6) shows that such averaging gives an accurate result for an elastically uniform body only, the auxiliary system of stress for such a body reducing to $\tau'_{ij} = \delta_{1i}\delta_{1j}$. This result refers to single crystals and to polycrystals built up of isotrope crystals. In general, the calculation of the magnetostriction involves the calculation of the non-uniform auxiliary system of stress, which the complicated irregular structure of polycrystals permits to make only in an approximate manner. The equation (6) of itself represents an exact consequence of the assumptions concerning the forces that give rise to magnetostriction.

As an example of the use of the equation (6) in a general case, we shall calculate the longitudinal magnetostriction of polycrystalline iron between the de-

magnetized state and saturation. Both the elastic properties and magnetostriction of iron crystals are strongly anisotropic; and hence a considerable difference in the results may be expected as compared to the simple averaging.

We determine the stress τ'_{ij} assuming that the crystals are of the spherical form. We replace the environment of each crystal by an indefinitely extended uniform isotrope solid characterized by the same values of the constants of elasticity as the polycrystal as a whole. The stresses at infinity (i. e. at a distance large compared with the dimensions of the crystal) are given by the boundary conditions that determine the auxiliary system of stress. These assumptions, taken together, specify a definite problem of the theory of elasticity. The approximate solution of that problem by Ritz' method, assuming uniformity of stress within the crystal, gives in a system of coordinate axes parallel to the principal axes of the crystal

$$\tau'_{ij} = \begin{cases} \frac{(c_{11}-c_{12})C_1}{\mu} l_i l_j & (i = j) \\ \frac{c_{44}C_2}{\mu} l_i l_j & (i \neq j) \end{cases} \qquad (7)$$

where

$$C_1 = \frac{(6\lambda+16\mu)(6\lambda+21\mu)-36(\lambda+\mu)^2}{(6\lambda+16\mu)(6\lambda+11\mu+5c_{11}-5c_{12})-36(\lambda+\mu)^2},$$

$$C_2 = \frac{(6\lambda+16\mu)(6\lambda+21\mu)-36(\lambda+\mu)^2}{(6\lambda+16\mu)(6\lambda+11\mu+10c_{44})-36(\lambda+\mu)^2},$$

λ, μ, c_{11}, c_{12}, c_{44} are the constants of elasticity, $\mathbf{l}$ is the unit vector in the direction of the measurement, C_1, C_2 are the coefficients relating the components of strain within and outside the crystal. We omit the calculations leading to (7) and propose to give them in a more detailed communication.

For $e_{ij}^{(m)}$ in the state of saturation in the direction $\mathbf{l}$ we have

$$e_{ij}^{(m)} = \begin{cases} \left(\frac{3}{2} l_i l_j - \frac{1}{2}\right)\lambda_{100} & (i = j) \\ \frac{3}{2} l_i l_j \lambda_{111} & (i \neq j) \end{cases} \qquad (8)$$

where λ_{100} and λ_{111} are constants.

In the demagnetized state the directions of the spontaneous magnetization at different points of each crystal are distributed with equal probability along its principal axes, and the values of $e_{ij}^{(m)}$ averaged throughout the volume of the crystal are therefore zero.

Averaging in (6) the expression under the integral sign over different orientations of the crystal, i. e. over different $\mathbf{l}$, we obtain

$$\frac{\delta l}{l} = \frac{1}{5}\lambda_{100}\frac{(c_{11}-c_{12})C_1}{\mu} + \frac{3}{5}\lambda_{111}\frac{c_{44}C_2}{\mu}. \qquad (9)$$

The values of the coefficients at λ_{100} and λ_{111} for iron are 0.293 and 0.707, respectively, the constants of elasticity being taken as $c_{11}=23.7$, $c_{12}=14.1$, $c_{44}=11.6$ ([3]); $\lambda=9.97$, $\mu=8.24$ ([4]) in 10^{11} dyne/cm^2 units. We neglect here the magnetomechanical effects affecting the measurements of these constants. The sum of the coefficients is equal to unity, which is only natural, because, putting $\lambda_{100}=\lambda_{111}=\frac{\delta l}{l}$ we obtain an approximate relation between λ, μ and c_{11}, c_{12}, c_{44}.

The values of $\frac{\delta l}{l}$ obtained with different data for λ_{100} and λ_{111} are given in the table. The data assumed to be the best by Becker and Döring ([8]) (bottom line) are in good agreement with Kornetzki's experimental value: $\frac{\delta l}{l} = -8\times10^{-6}$ ([10]).

λ_{100}	λ_{111}	$\frac{\delta l}{l}$	$\frac{\delta l}{l}$ un. stress	$\frac{\delta l}{l}$ un. strain
25.5[5]	—18.8[5]	—5.8	—1.1	—10.0
26.5[9]	—19.2	—5.9	—0.9	—10.1
17 [6]	—18.8[5]	—8.3	—4.5	—12.0
19.5[7]	—18.8[5]	—7.6	—3.5	—11.4

Note. All the values are multiplied by 10^6.

For comparison there are listed the values calculated from equation (6) under simplified assumptions about the auxiliary system of stress τ_{ij} (uniform stress and uniform strain, correspondingly). The first of them gives an equation coinciding with that of Akulov[2]

$$\frac{\delta l}{l_{\text{un. stress}}} = \frac{2}{5}\lambda_{100} + \frac{3}{5}\lambda_{111}, \tag{10}$$

the second gives

$$\frac{\delta l}{l_{\text{un. strain}}} = \frac{1}{5}\lambda_{100}\frac{c_{11}-c_{12}}{\mu} + \frac{3}{5}\lambda_{111}\frac{c_{44}}{\mu}. \tag{11}$$

Analogous to the magnetostriction, the irregular structure of polycrystals must affect the mechanism of the galvanomagnetic and thermomagnetic phenomena. In the case of cubic crystals, however, the conductivity, as well as any other tensor of the rank two, is isotropic and the calculation of the galvanomagnetic effect for polycrystals is reduced to a simple averaging throughout the volume of the body. The comparison with the experiment shows that the equations of type (10) [2], thus obtained, give the best result expressly in the case of galvanomagnetic effect.

Lomonossov State University.
Moscow.

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PHYSICS

IS THE PARTICLES' «SELF-FIELD» A PHYSICALLY OBSERVABLE QUANTITY?

By M. MARKOV

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In the mathematical apparatus of modern electrodynamics (Dirac's equation, quantized electromagnetic field) the expression for the electrostatic self-energy of the electron is a superfluous element. Here the principal difficulty does not lie in the divergence of the respective expression, but rather in that the discussed expressions, even if they were finite, could not, within the scope of Dirac's equation, replace the term $\beta_4 mc^2$ connected with the self-energy of the electron.

Surely, the equation of Dirac, unlike classical or non-relativistic equations of motion, is not deprived of mathematical sense by the elimination of the term $\beta_4 mc^2$; nevertheless, the obtained equation, should it even involve a finite term connected with the electromagnetic self-energy of the charge, has no connexion with the electron. But the principal thing, as has been pointed out by this author earlier (1), is that a system of equations of quantum electrodynamics, written down consistently in the relativistic form (electrodynamics of Dirac—Fock—Podolsky), leaves no room for an electromagnetic interpretation of the electron's self inert mass, because the equation of motion for the particles contains no expressions of the type $\int \frac{E^2+H^2}{8\pi} d\nu$ relating to the energy of the field.

Of course, the question can be treated in a broader way, viz. one may attempt deriving the equations of motion for the electron (Dirac's equation type) from the equations of motion for the field. But such a consequent treatment of the problem is difficult to discuss, because no attempt in this line has ever been reported in literature. At this juncture it is difficult to tell whether such a consecutive program has any physical sense in the domain of the quantum theory, because its physical contents is very meagre and almost equivalent to the trivial statement that between the expressions for the energies and the mass there is a universal relation. The presence of particular properties in the electron (spin), not reducible to the relation between energy and mass, presents essential difficulties for this program to be carried out. Moreover, the considerations connected with the fact that the equations of electrodynamics written down in strict correspondence for the relativistic case involve different times, are very general. It would seem that in any electrodynamics different times should be introduced for different particles: hence a particular time for the field; hence an equation for the field without the right-hand member (D. F. P.). But in such an electrodynamics

the particles and the field are again to be treated as co-existent and interacting systems, not reducible to each other.

Turning back to Dirac's equation, it is well to emphasize the circumstance that in the computation of all observable effects the expressions connected with the electromagnetic self-energy are ignored, and this leads to no collision with the experiment. The «superfluous» nature of the expressions connected with the particles' «self» electromagnetic field justifies an attempt to analyse the notion of «self-field» for its physical meaning, that is to prove, whether it may or may not be connected with a physically observable value.

The intensity of the «field» (E) about the electron (e) is measurable, in principle, as the force of interaction of the electron with the test charge (e_p).

$$E = -\frac{ee_p}{r^2_{ee_p}} \cdot \tag{1}$$

In analysing the question the conception of «field» is used, viz. it is supposed that the test-body only reveals the presence of such a «field of forces» localized in a vacuum, while by itself it has no relation in principle to the creation of such a state in the surrounding space.

It is supposed that this state of the space retains its physical meaning after the removal of the test-charge, and that the field is capable of a reverse action upon its very «source», this action proceeding according to the same law (1).

Obviously, there is a weak point in this logical chain of conclusions, since the existence of the «self-field», of a «field» acting upon its source, is based on a hypothesis not corroborated by a physical experiment.

True enough, an electrically charged body in the macrocosm enables one to speak of the «self-field» or «self-energy» of a given charge, for it is possible to find the forces and energy acting between the electrons that charge the body. But to suppose an interaction between separate parts of an electron is to draw an analogy which has no real foundation for the present.

In other words, we arrive at the conclusion that the superfluous expressions in modern electrodynamics are not invested with the character of the usual observables.

It is therefore advisable to construct a theory without these physically superfluous elements. In such a theory the test-body should, in accordance with the above considerations, enter organically into the equations of the field. To put an example, Poisson's equation for the potential of a resting charge should be written in the form

$$\frac{\partial^2\varphi}{\partial x_p^2}+\frac{\partial^2\varphi}{\partial y_p^2}+\frac{\partial^2\varphi}{\partial z_p^2} = -4\pi e e_p \delta(r-r_p), \tag{2}$$

where e is the charge of the electron; e_p, the charge of the test-body; r, the coordinate of the electron; r_p, the coordinate of the test-charge.

The solution of equation (2) is

$$\varphi = \frac{ee_p}{r_{ee_p}} \tag{3}$$

and, finally, for the field at the point r_e we have

$$E_{ep} = -\operatorname{grad}_{rp}\varphi = -\frac{ee_p}{r_{ee_p}} \cdot \tag{4}$$

Equation (2) and its solution (3) point only to the existence of a definite interaction between two different charges and give no indication of any action of the charge upon itself *. Only the external field should therefore enter into the equation of motion for the charge.

* For $r_{ee_p} \to 0$ we have $\varphi \to \infty$, but the charge of the system is, as before, equal to $e - e_p$ rather than e.

Naturally enough; it is the construction of a consistent theory of the field as a part of the quantum theory that is of interest, and not merely the electrostatic case.

Here we have, on the one hand, Dirac's equation (with quantized electromagnetic field), from which the treatment of the problem usually starts, and, on the other hand, we have the computation scheme of the perturbation theory, which is used in practice for the calculation of observables. It is known that on closer consideration Dirac's equation in this case proves to represent the trivial relation (2)

$$\frac{\partial \psi}{\partial t} = \infty$$

owing to the presence of divergent expressions in the «equation».

In other words, the «computation scheme» of the theory of perturbation is the sole working apparatus of the theory whose reliability is checked by the physical experiment.

According to its mathematical meaning, the «perturbation theory» must give approximate expressions for the solution of Dirac's equation. But as the latter has virtually no solution at all, we are compelled for the present to use the mathematical apparatus of the «perturbation theory» as an independent computation scheme. in no way connected with the equation of Dirac.

Some time ago we discussed the possibility of legalizing this state of affairs by means of a clear and logically consistent reasoning (3). This means that we must rigorously define the physical notions involved in the given «computation scheme», to define the operations they are subjected to in an inwardly consistent manner, and in that way to obtain an orderly developed computation scheme for different electrodynamic processes, free from the notorious difficulties of electrodynamics.

The physical notions of the system considered are a free electron with inert mass (m) and free photons, longitudinal and transverse. Within this scheme all electrodynamic properties are reduced to the emission and absorption of photons by the electron. Every electrodynamical effect is brought into being through a chain of photon emission and absorption processes.

Every such chain (H_{AB}) transfers the system from a state A to a state B. The rules for composing the matrix element and cross-sections can be defined directly. Making use of the relativistic theory of perturbation as developed a few years ago by Steckelberg, the «chains» can be written in a relativistically invariant form. Within the frame of such a mathematical apparatus the action of an electron upon itself is materialized by a chain consisting of the emission and subsequent absorption of the same photon. If we adopt the restriction that only the elementary chains giving observable modifications of the system are materialized, then chains of the type H_{AA} will prove to be forbidden, because the initial and the final state are not distinguishable from each other*.

Perhaps, it will prove necessary in the subsequent to impose stronger restrictions upon the selection of materializing chains. Indeed, the interaction between the charges is realized by the chain in which one charge emits a photon and the other absorbs it. One may conceive chains in which such an exchange involves two, three and more photons (both transverse and longitudinal). These chains lead to an interaction law, inversely proportional to the higher order of distance between the charges. A law higher than the second order could lead to a catastrophe (for instance, in a hydrogen atom). It is quite possible that such chains should also be forbidden by the generalization of the previous restriction, e. g. in the form: Processes that take place in reality are realized by means of chains of minimum length only.

From the point of view of minimum principles materialized in nature such a restriction seems to be rather natural.

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* In the electrostatic case this statement is equivalent to the meaning of the equation (2).

In a recent work by Heitler and Peng (4) in which essentially similar ideas are evolved, the restriction of the perturbation theory is formulated as follows: «For each pair of states A and B with the same energy, the matrix element H_{AB} is to be defined as that of the lowest order only, while all higher order matrix elements for the same transition are to be omitted».

There have been many attempts, some very tempting and ingenuous, to solve the well-known difficulties of the theory of interaction between the particles and fields, but any of them, even if successful, would give only a limited solution of the problem. The direction taken in this paper is to treat the problem in a very general manner for any field and particle, and it gives a final solution in the sense that it evolves a general method for the calculation of any practically interesting effect, and this method is free from inner contradiction.

From a general point of view, this trend implies the abandonment of the idea of establishing a connexion between the particles' own masses and the corresponding fields. This idea seemed very promising at the time when the electromagnetic outlook was in its zenith, but lost its original monistic finality when it had become necessary to discuss new forces of interaction between elementary particles, hence new fields, and consequently, new additions to the proper mass.

It becomes increasingly obvious that this point of view does not lead to a theory of elementary particles, since the latter requires some general approach to the proper masses of all elementary particles.

The direction discussed here is unable to solve the question connected with the masses and properties of elementary particles. The masses of all particles are treated in a unique manner as inert masses. This leaves room for a single point of view, but all these are questions for fundamental theories to come. Roughly speaking, these theories will have to be considered as analogues of the generalized theory of relativity in the macrocosm, the inert masses of elementary particles being the subject of their study.

P. N. Lebedev Physical Institute.
Academy of Sciences of the USSR.

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PHYSICS

THE SPECTRUM OF THE SOFT COMPONENT IN AIR AT HIGH ENERGIES

By I. POMERANCHUK and A. KIRPICHEV

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The spectrum of the soft component due to the decay electrons and making up the main part of the soft component of the air at small altitudes has been studied by Heisenberg and Euler (1) and by Tamm and Belenky (2). However, Heisenberg and Euler made use of an imperfect form (3,4) of the cascade-theory of showers. After having been improved by Landau and Rumer (5), the apparatus of this theory may be applied for obtaining by far more accurate results. Instead of evaluating precisely the meson spectrum, Tamm and Belenky analysed mesons of a certain mean energy. Such a procedure cannot be justified when dealing with the spectrum of the soft component with E energies higher as compared to the critical ε. It is obvious that the high energy soft component can arise only as a result of disintegration of high energy mesons; the form of the spectrum of the soft component within this energy range is therefore greatly dependent on the kind of the spectrum of mesons, in case they are of high energy.

The analysis of the spectrum of the soft component with $E \gg \varepsilon$ is of particular interest in connexion with the study of transition curves for great thicknesses accessible only to showers that are due to soft particles of the air with $E \gg \varepsilon$. We have plotted transition curves on the basis of the spectrum of the soft component found in this work.

In determining the spectrum sought for, we took as a basis the spectrum of electrons of decay that arise as an immediate result of meson disintegration:

$$\Phi(E_0)\,dE_0 = \frac{X_0 \mu C^2 A \gamma dE_0}{C\tau_0} \int_{E_0}^{\infty} \frac{dW}{W^2 (W + W_0)^{\gamma+1}}, \qquad (1)$$

where τ_0 is the time of decay of a meson at rest ($\tau_0 \approx 2.5 \times 10^{-6}$ sec.) (6); X_0, cascade unit of length in air; μ, mass of meson at rest; C, velocity of light; W, energy of the meson; W_0, energy spent by the meson on ionization when travelling from the point of creation to the point where it is observed ($W_0 \approx 2.2 \times 10^9$ eV for sea level in the case of vertical direction; $W_0 = \frac{aT}{\cos\vartheta}$, where ϑ is the angle formed by the direction of the mesons motion and the vertical; $a \approx 2.2 \times 10^6$ eV; T, depth of water in cm).

According to Heisenberg and Euler (1), the differential spectrum of mesons is taken by us in the shape

$$\frac{A\,dW\,\gamma}{(W+W_0)^{\gamma+1}}$$

where A is a constant, $\gamma \approx 2$. The decay of mesons is not taken into account here, which seems consistent, since we shall be interested in mesons with high energies, in the case of which the spectrum is not materially affected by decay. In this connexion it should be noted that the evaluation of the decay of mesons involves no special difficulties (1). The spectrum of the soft component $F(E)$ can be found with the aid of (1):

$$F(E)=\int_E^{\infty}\Phi(E_0)\,dE_0\int_0^{\infty}a(E_0,E,t)\,e^{\varphi(E_0,E,t)}\,dt; \qquad (2)$$

ae^{φ} denotes as usual (5) the number of particles with energies higher than E in a shower formed by the initial particle with energy E_0, the shower being supposed to have travelled a path t (t being measured in cascade units). $F(E)$ evidently denotes the integral spectrum—the number of particles with energies higher than E.

As ae^{φ} is the only magnitude dependent on t, we shall in the firse line carry out integration along t. The chief part is played thereby by such t for which the number of particles with energies higher than E attains the maximum ($s=1$) (7):

$$\int_0^{\infty}ae^{\varphi}dt=a(s=1)\,e^{\varphi(s=1,E_0,E)}\sqrt{-\frac{2\pi}{\lambda_1''}}=B(1)\,\frac{E_0}{E}\,. \qquad (3)$$

Here use has been made of the correlation: $\varphi(s=1)=\ln\frac{E_0}{E}$ (5), a being a slowly varying function as compared to e^{φ}.

$$F(E)=\int_E^{\infty}\Phi(E_0)\frac{E_0}{E}B(1)\,dE_0=\frac{1}{E}\int_E^{\infty}E_0B(1)\,dE_0\int_{E_0}^{\infty}\frac{\mu C^2N_0dW}{rz_0W^2}f(W); \qquad (4)$$

$f(W)$ is the differential spectrum of mesons [it must not necessarily coincide with that made use of in (1). In particular, it may account for the decay of mesons].

Let us now change the order of integration in the double integral (4):

$$F(E)=\frac{N_0\mu C}{Erz_0}\int_E^{\infty}f(W)\,dW\int_E^{W}E_0B(1)\,dE_0=\frac{N_0\mu C}{2Erz_0}\int_E^{\infty}dW f(W)\,B(s=1,\,W,E). \qquad (5)$$

It should be noted here that $a(E,E)=0$, the value of the inner integral along E_0 is determined by the upper limit. Now let us make use for $f(W)$ of the expression

$$f(W)=\frac{A\gamma}{(W+W_0)^{\gamma+1}}$$

$$F(E_0)=\frac{A\gamma N_0\mu C}{2Erz_0}\int_E^{\infty}\frac{dWB}{(W+W_0)^{\gamma+1}}\approx\frac{A\gamma N_0\mu CB}{2Erz_0(rE+W_0)^{\gamma}}$$

$$r>1,\quad r\approx 1,\quad r=\text{const}.$$

As B is almost independent of energy (except the case when $W\sim E$), we can take it outside the sign of integral. For charged particles B is equal to (8):

$$\frac{0.137\sqrt{1.56\cdot 2\pi\left(\ln\frac{W}{E}-1\right)}}{\sqrt{\ln\frac{W}{E}-0.37}}=0.4.$$

For photons B is equal [9] to $\frac{9}{7}\,0.4\approx 0.5$.

Ultimately, making use of the correlation for the full charged soft component F_0 due to electrons of decay, we find [1,2]

$$F_e=0.4F_0\,\frac{\varepsilon}{E}\,\frac{1}{\left(1+\frac{r_1E}{W_0}\right)^{\gamma}},\qquad F_\gamma=0.5\,\frac{\varepsilon}{E}\,\frac{1}{\left(1+\frac{r_2E}{W_0}\right)^{\gamma}} \tag{6}$$

$$\frac{F_0}{N_0}=\frac{\mu CX_0}{2\tau_0\varepsilon}, \tag{7}$$

where γ are quanta; e, electrons and positrons; N_0, the total number of mesons. Relation (7) may easily be obtained by applying the law of energy conservation, on the assumption that half of the energy of the mesons is carried away by the neutrino.

As seen from (5), $F(E)$ is directly expressed through the integral spectrum of the mesons $\int_E^\infty f(W)\,dW$, B being constant almost everywhere except the case $W\approx E$. Making use of (5), we are able to obtain the spectrum of the soft component, for example, in the case when it is necessary to account for the mesons of decay in the spectrum.

An accurate computation of the coefficient r involves considerable difficulties, requiring the analysis of an undeveloped shower with a small number of particles we are interested in.

With $E\gg W_0$ in the integral along W in (5) the main part belongs to the region $W\approx E$, i. e. the chief rôle is played by showers consisting of a small number of particles endowed with the necessary energy $\geqslant E$. Hence it follows that within this region of energies the numerical value of the coefficients 0.4 and 0.5 with $E\gg W_0$ cannot be accurate (for the same reason which prevents the calculation of r).

From (6) we may infer the presence of an appreciable number of particles with $E\gg\varepsilon$. For instance, putting $E=4\varepsilon$, we shall obtain with $r_1=r_2=2$ at sea level, $\gamma=2$:

$$\frac{F_e+F_\gamma}{F_0}=\frac{0.9}{4\times 1.96}=11.5\%;$$

with $r_1=r_2=\frac{3}{2}$

$$\frac{F_e+F_\gamma}{F_0}=\frac{0.9}{4\times 1.7}=\frac{0.9}{6.8}=13\%.$$

It can be said that the number of particles with energies higher than 4ε makes up at sea level a magnitude of the order of 12—13% to the total soft charged component due to the decay of mesons.

The method here applied may easily be extended to the soft component due to δ-electrons.

V. M. Molotov University. Erevan.
P. N. Lebedev Physical Institute.
Academy of Sciences of the USSR.

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CHEMISTRY

CATALYSIS IN REDUCING POTASSIUM AND SODIUM SULPHATES FROM CHLORIDES DECOMPOSED WITH SULPHUR DIOXIDE IN THE PRESENCE OF OXYGEN

By S. I. WOLFKOVICH, Corresponding Member of the Academy, and F. G. MARGOLIS

Of the different methods for producing potassium sulphate, particularly attractive seems to be the possibility of decomposing potassium and sodium chloride with sulphur dioxide in the presence of air and water vapour, according to the over-all equation:

$$2KCl + SO_2 + H_2O + O \rightarrow K_2SO_4 + 2HCl \qquad (I)$$

As is known, a similar process was proposed and realized in the eighties of the last century by Hargreaves and Robinson for the production of sodium sulphate. This process was carried out at several German and English plants in a very bulky apparatus, because it proceeded exceedingly slowly—15 to 20 days at as high a temperature as about 500° C. Its advantage in point of economy lies in the cheapness of the raw material. This is sodium chloride and sulphur dioxide evolved in vast quantities as refuse at many copper-smelting and other metallurgical plants. The process was carried out in a periodically working apparatus—large towers charged with cakes that presented a mixture of the salt with sawdust. It has aquired no extensive application, however, on account of its long duration, and is at present abandoned almost everywhere.

Now we have set ourselves the task of trying the Hargreaves-Robinson method in the production of potassium sulphate. The justification of such an attempt lies in the dependence of agriculture upon this product as a fertilizer, and in the fact that there is also a need for it in different industrial fields. It was, moreover, our object to investigate the usefulness of a number of catalysts for sharply accelerating the technological process. The possibility of obtaining potassium sulphate without using sulphuric acid, the production of which requires a bulky and extensive equipment and the existence of vast, as yet non-utilized reserves of sulphur dioxide in the non-ferrous metallurgy, justified the expectation that the process would prove economic provided it has been sharply accelerated.

The first laboratory experiments (1) confirmed the possibility of accelerating this process some hundreds of times with the aid of catalysts containing ferric oxide. Our further investigations were carried on with the aim of finding the most easily obtainable and cheap catalysts, bringing greater precision into our knowledge of the physical-chemical conditions of the process when working at a low concentration of hydrogen sulphide, with the production of hydrochloric acid; and of determining the general features of the apparatus to be used in the technological process.

According to the data of several foreign authors ([3–5]), the process of converting KCl into K_2SO_4 takes place in two principal steps:

$$SO_2 + 2KCl + H_2O \rightarrow K_2SO_3 + 2HCl \quad (II)$$

$$K_2SO_3 + O \rightarrow K_2SO_4, \quad (III)$$

i. e. the process takes place through the formation of the sulphite.

The present investigation and calculations of the equilibrium constant of the reaction (II) according to Nernst, as well as a preceding study of the processes of oxidation of sulphites ([6]), show that the process follows a different scheme:

$$SO_2 + O \rightarrow SO_3 \quad (IV)$$

$$SO_3 + H_2O + 2KCl \rightarrow K_2SO_4 + 2HCl, \quad (V)$$

i. e. the oxidation reaction proceeds in the gaseous phase; potassium chloride is reacted upon not by SO_2, but by SO_3 in the presence of water vapour.

A great influence upon the course of the reaction is exercised by the concentration of sulphur dioxide; and a lesser influence, by the concentration of oxygen. In practice, the reaction (I) proceeds slowly up to 400—450°, and upon raising the temperature to 550—600° it is rapidly accelerated. The process is also greatly accelerated by the presence of moisture.

The choice of catalysts for this process is limited by a number of practical requirements: the catalyst should be not only cheap and easily available, but also harmless (especially for plants), since it is carried out with the salt; it would be hardly possible practically to separate and reclaim it. The catalyst should be as active as possible in order to reduce its additions to the salt to a minimum, to prevent a contamination of the product. Moisture should not affect it.

We introduced the catalyst into the original salt in the shape of powder in an amount up to 1% to the weight of the salt. In the first investigation we used a number of catalysts: Fe_2O_3, $Fe(OH)_3$, Al_2O_3; CuO, $Co(NO_3)_2$; $CoSO_4$; pyritic scoria, an alloy of clay with the scoria, metallurgical slags from smelting cast iron from titaniferous magnetites and other ores.

In view of the high recoveries of K_2SO_4 and a great acceleration of the process in the presence of catalysts containing ferric iron, the latter, particularly pyritic scoria, titaniferous magnetite, and the products of their acid and alkali treatment, were subjected to a detailed investigation. The most favourable results were obtained with pyritic scoria.

In the experiments, a study was made of the effect upon the rate of the process and the recovery of potassium sulphate of the following factors: the duration of the reaction; the temperature conditions; the concentration of SO_2 in the gas; the volume velocity of gas; the height of the layer and the degree of reduction of potassium chloride; the amount of catalyst, etc.

In the optimum experiments, the electric furnace was charged with a mixture of finely ground KCl with 1% scoria. The SO_2 content in the air mixture was 5.5—6.5%. The temperature of the reaction was maintained at 520—550° C. In a number of experiments the salt was mixed also with wood sawdust which burn out, contributing to the formation of a porous mass and thereby to a higher recovery of K_2SO_4. A slight moistening of the salt favoured a higher recovery.

That the process follows the equations (IV) and (V), and not (II) and (III), was confirmed also by the fact that the catalysts, iron scoria in particular, did not appreciably accelerate the reaction of oxidation of solid potassium sulphate.

Since the heterogeneous process studied consists of a complex chain of reactions occurring at a high temperature; it is necessary that a good contact of gases with the solid phase and diffusion be secured, as well as a constant renewal of the surface of the solid reagents and a uniform distribution of the catalyst. Although the reaction of potassium sulphate formation is an exothermal one, the

amount of the heat evolved is nevertheless insufficient for the process to go on without any heat supply from outside. Therefore the set-up to be used in the process requires a proper heat regime in the reaction apparatus (furnace) to be secured. A number of foreign patents ([2]) recommend different procedures and apparatus for a rationalization of the process; however, all of these do not solve the chief problem—securing a rapid and continuous process by means of catalysis, and have not received practical application.

We have tested several models of periodical and continuous reaction apparatus: externally heated vertical and inclined tubes with rotating paddles; network drums, between which potassium chloride is circulated, through which hot gas is passed, etc.

If the problems of equipment have been satisfactorily solved, of interest will be the treatment with sulphur dioxide not only of pure potassium and sodium chlorides, but of normal salts; as well: sylvinite (KCl, NaCl) and carnallite ($KClMgCl_2.6H_2O$) with the production of corresponding sulphate mixtures. A mixture of potassium and magnesium sulphates may be of considerable interest not only as a fertilizer, but also as a flame extinguisher, and a semiproduct for obtaining a number of other technical products.

If a satisfactory development of the apparatus for the process be reached, great interest may be presented by a modification of the process distinguished by the absence of water vapour, according to the equation:

$$2KCl + SO_2 + O_2 = K_2SO_4 + Cl_2, \qquad (VI)$$

This reaction permits to obtain, along with hydrochloric acid, chlorine (or sulphuryl chloride) with a practically good recovery, and is at present investigated in the United States ([7]) and in the USSR—at the Scientific Institute of Fertilizers and Insectofungicides. Experiments with the use of iron catalysts for speeding up the reaction (VI), carried out in the above-named Institute by G. K. Boreskov upon our suggestion, have also confirmed the possibility of greatly accelerating it.

Conclusions. The present work has confirmed the possibility of accelerating the process similar to that of Hargreaves-Robinson some hundreds of times (from 16—20 days to 1—1.5 hours) by using catalysts containing ferric oxide.

One of the highly effective, readily available and cheap catalysts is pyritic scoria—a refuse of sulphuric acid manifacture. The optimum norm of this catalyst is about 1% from the weight of the salt used. The optimum temperature of the process is 500 to 550°C. The potassium sulphate recovery was 95—96%. The time of the reaction depends on the design of the furnace; in the present work it did not exceed 1.5 hours. The degree of utilization of SO_2 in optimum experiments amounted to 40% per one passage of gas (without circulation).

The problems concerning the apparatus, as well as the determination of the conditions of a more complete utilization of SO_2 and the HCl produced at a low concentration (up to 3—5%) necessitate further study.

Of great interest is also a further, more profound study of the reaction of decomposition of potassium and sodium chlorides with sulphur dioxide in the presence of oxygen, but in the absence of water vapour, with the production of chloride and sulphuryl chloride.

Institute for Fertilizers and Insectofungicides.

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CHEMISTRY

THERMODYNAMICS OF CRYSTALLOCHEMICAL COMPOUNDS OF AUSTENITE

By N. V. GRUM-GRZIMAYLO

(Communicated by N. P. Chizhevsky, Member of the Academy, 23. III. 1943)

It has been shown in my previous paper [1] that the departure from Wegardt's rule for the dimensions of the crystalline lattice constant is connected with the presence of crystallochemical compounds in the solid solution. The study of iron alloys with different elements has proved that within one phase several crystallochemical compounds are possible. Consequently, their interaction should take place.

The coefficient of proportionality determining the numerical value of the departure from Wegardt's rule depends upon the internal energy of the crystallochemical compound and is connected with it by a simple formula. It is possible, therefore, starting from the definition of the constant of the crystalline lattice and knowing the chemical formula of the compounds, to determine its internal energy and entropy. These data suffice for constructing the equation of the isochore, and by means of it the equilibrium between the ordered and disordered phases may de determined.

For iron-carbon alloys the data by Esser and Müller were shown earlier to have served for determining the following formulae of the constant at 800 and 1100° C:

$$\pi_{800^\circ} = 3.5640 + 4.3386C - 4.3943\,Fe^3C - 1.1965\,Fe^{18}C^6 \cdot 10^6$$

$$\pi_{1100^\circ} = 3.6600 + 5.4893C - 5.3776\,Fe^3C - 1.3220\,Fe^{18}C^6 \cdot 10^6$$

where Fe denotes the atomic concentration of iron, and C, the atomic concentration of carbon.

According to (2), the formula for the coefficient of the constant is:

$$A = \frac{\rho kT}{3p_T \pi_0^2} e^{-\left(\frac{u}{kT} + \frac{c}{k}\right)} = \frac{4 \cdot 1.38 \cdot 10^{-16} T}{3p_T \cdot 9.81 \cdot 10^5 \pi_0^2} e^{-\frac{4.182 \cdot 10^7 (m+n)}{1.38 \cdot 10^{-16} \cdot 6.02 \cdot 10^{23}} \left(\frac{u}{T} + \frac{c}{}\right)} =$$

$$= 1.062 \cdot 10^8 \frac{T}{p_T \pi_0^2} e^{-0.5033 \left(\frac{u}{T} + c\right)(m+n)}$$

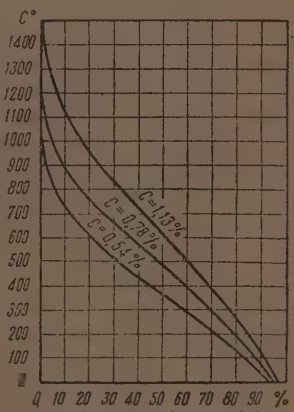
where T is absolute temperature: u, internal energy; σ, entropy at absolute zero; p_T, modulus of elasticity; π, the constant of the pure solvent; m and n are the indices in the formula of the compound.

For the modulus of elasticity of iron at high temperatures following formula has been given by Dorfman:

$$p_T = 3.4e^{\frac{1050}{1580-T}}.$$

And it is proceeding from these data that we will here compute the value of the internal energy and entropy at 800 and 1100° C.

Assuming, for orientation, a parabolic dependence between the internal energy and temperature, and taking into account that the internal energy changes its sign (i. e. is equated to 0) at the transformation temperature of the alloy A_1, it is possible to determine the formulae of internal energy for the two compounds, as well as their entropy at absolute zero:



For graphite system ($Fe_{18}C_6$): internal energy $u_g = -4497 + 0.004425\ T^2$; entropy at absolute zero $\sigma_g = -67.12$.

For carbide system (Fe_3C): internal energy $u_c = 1072.5 - 0.001686T^2$; entropy at absolute zero $\sigma_c = -9.210$.

For disordered phase: internal energy $u_p = 4774 + 0.004817\ T^2$; entropy at absolute zero $\sigma_p = -48.74$ *.

The equation of the isochore can in our case be taken in the following form:

$$\log K = -\left[\frac{a}{4.571\,T} + \frac{bT}{4.571}\right] + \frac{\sigma + n\sigma_p}{4.571},$$

where n is equal to 1 and to 6 for the carbide and graphite system, respectively. Substituting these values, we finally obtain:

For the graphite system

$$\log K_g = +\frac{983.8}{T} - 0.000968\ T + 59.277. \qquad (1)$$

For the carbide system

$$\log K_c = -\frac{234.6}{T} + 0.0002365T + 8.645. \qquad (2)$$

Here the equilibrium constant is: $K_g = \frac{C_{Fe_{18}C_6}}{C_{disord}}$ and $K_c = \frac{C_{Fe_3C}}{C_{disord}}$ where $C_{Fe_{18}C_6}$ and C_{disord} are atomic concentrations of the graphite molecules and random distribution of carbon atoms; C_{Fe_3C} are atomic concentrations of carbide molecules. These equations show in the first place that the concentration of the disordered phase is very low. If it is neglected, we have $C_{Fe_{18}C_6} + C_{Fe_3C} = C$, where C is total content of carbon in the alloy (in at. per cent). Putting $x = \frac{C_{Fe_{18}C_6}}{C}$, we obtain: $1 - x = \sqrt[6]{\frac{K_c}{K_g}}\sqrt[6]{\frac{x}{C^5}}$, and from this equation, considered jointly with (1) and (2), one may calculate the relative content of graphite and carbide molecules in the alloy.

* For the graphite system ($Fe_{18}C_6$) A_1 is put at 735° C; for the carbide system (Fe_3C) 723°C for a disordered system—707°C (tentatively).

We have also extended these calculations to temperature at which austenite is only observed in the supercooled condition.

On the adjoined graph the results of these computations are presented.

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GEOLOGY

GEOLOGICAL CONDITIONS OF GAS ACCUMULATIONS IN COAL DEPOSITS OF THE KUZNETSK BASIN

By A. I. KRAVZOV

(Communicated by P. I. Stepanov, Member of the Academy, 7. V. 1943)

The geological investigations carried out by the Institute of Geological Sciences of the Academy of Sciences of the USSR in three large regions of the Kuznetsk basin (Kemerovo, Stalinsk and Leninsk-Kuznetsk regions) permitted to establish the geological conditions of gas accumulations in the coal deposits of the Kuznetsk basin. Especially representative in this respect is the Stalinsk region (Aralichevo coal deposit), with the discussion of which the present paper is concerned. This deposit was studied in 1923—1941 by the geologists V. I. Yavorsky, S. V. Kumpan and V. E. Nekipelov ([2,4,5]). From their data, the geological structure of the deposit appears as follows.

Stratigraphy and Lithology. The productive formation of the Aralichevo coal deposit is composed of rocks of the Balakhna series and is conformably overlain by rocks of the Non-coal (Bezugolny) series.

In its composition the coal-bearing formation is characterized by alternating sandstones, aleurolites, argillites and carbonaceous shales (Fig. 1). Within it up to 12 working coal seams have been encountered, having a thickness from 1 to 8.4 m, with an average above 2.5 m. The thickness of the series within the area prospected is 375 m. Coal saturation is up to 10%. The coal of the Aralichevo deposit belongs to lean coals, with a 7 to 12% volatile matter content, and is used as energetical fuel.

Exposures of country rocks within the productive formation are relatively rare, and the structures of this particular deposit have been reproduced mainly from data of prospecting.

Tectonics. From the materials obtained, the structure of the Aralichevo deposit appears as brachyanticlinal folds: in the West the Aralichevo brachyanticline, and in the East the Redakovo-Sokolukha (Fig. 2). The Aralichevo comprises in the South the Teshev and Kopylikha anticlines: and the Redakovo-Sokolukha, the Gorbunikha, Redakovo and Sokolukha anticlines. The formation of those folds refers to the Cimmerian phase.

These folds were formed as a result of tangential stress which proceeded, according to Butov, Yavorsky and Ussov ([1,3]), from the side of the Salair range, i. e. in a direction from West to East. The formation of the folded structure produced disjunctive disturbances of the rocks composing the Aralichevo coal deposit. All the disjunctive disturbances are represented by conformable and unconformable upthrusts and overthrusts, frequently dying out with depth and along the strike. The majority of the disjunctive disturbances are contemporaneous with the formation of folding, since they mostly have a direction concordant with the folding and follow complementary folds.

Gas Content of Coal Seams and Accumulation of Gases in Anticlinal Structures. The investigations of the Institute of Geological Sciences and the Mining Institute of the Academy of Sciences carried on in 1942 showed all the coal seams to be gas-bearing to some degree or other; all the coal mines of the region are therefore under conditions of a gas regime and belong to the category of insecure ones. The fact that the coals af the Aralichevo deposit of the Kusnetsk basin belong to lean coals, unlike those of the same subseries of the Balakhna series of the adjacent Prokopiev deposit shows that the process of carbonization in the former has continued longer than in the coals of the adjacent regions. This is in turn an evidence of the fact that the productive deposits had been overlain by a greater thickness of sediments during the period of coal formation at Aralichevo. The thickness of the overlying sediments protected the deposit against the loss of gas evolved in considerable quantities in connexion with the metamorphism of the coals to lean ones, under the influence of the pressure of superincumbent rocks and of the elevated temperature at depth. The highly complicated geological structure of the region made up of a number of small folds accounts for a highly variegated picture of gas content within indiyidual areas. Thus, e. g., at the Ordjonikidze mine, the gas regime of different areas is sharply different within the same coal seam.

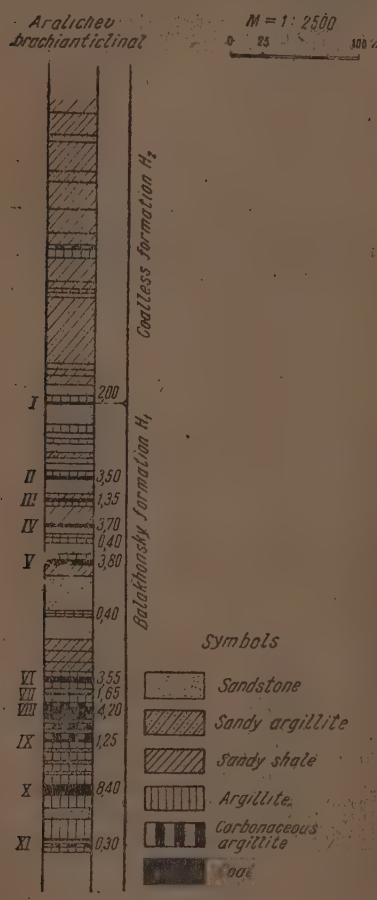

Fig. 1

It should also be noted that workings following the same fold show a different production of gas. On the gently sloping limb of the folds the evolution of methane is more intense than on the steep limb. The cause of this is that the migration of methane from the coal seams proceeds chiefly through the coal seams themselves and not through the overlying rocks. As a result, flatly lying coal seams show at the same depth from the day surface a greater gas-content than steeply dipping ones.

The gas content of the coal seams and hence the production of gas in the workings increases in the crest parts of anticlines and as one penetrates deeper into the subjacent horizons. The amount of gas produced in the workings is also determined by the close spacing of individual coal seams. In this respect all the coal seams worked in the Ordjonikidze mine may be subdivided into two groups. The first group comprises seams II—V; the second, VI—X.

Of the first group of upper seams the greatest gas production is shown by seams II and V. The production of gas in workings following seams III and IV to a considerable degree depends on how these workings proceed ahead of those following seams II and V. Thus, e. g., in the Ordjonikidze shaft, where the workings approached the crest portion within seam IV, the lava was for a time stopped by an abundant evolution of methane, although the rate of the air stream was brought to 3 m per second. Seam V is particularly rich in gas.

If we compare the amount of methane in shafts of different depth, it is found

gradually to increase with depth. Thus, in the Dmitrov shaft at the +205 m horizon there was no methane; the Redakovo shaft at the +210 m horizon showed traces; the Ordjonikidze shaft at horizon +165 m had up to 3% and up to 5% at the +85 m horizon. On this ground the horizons of the Redakovo shaft +170 and lower should be put outside any category.

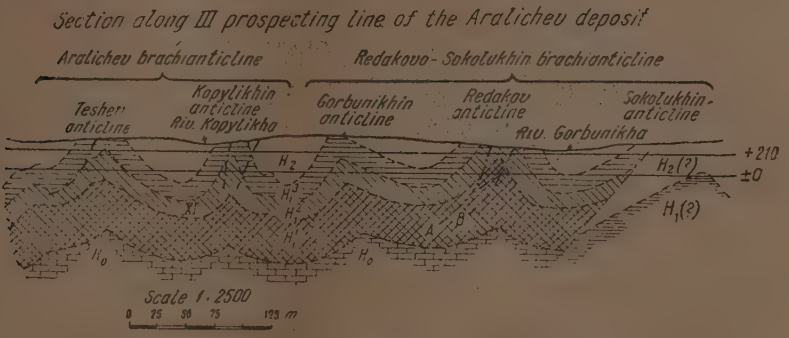

Fig. 2

A considerable increase in the gas content in the coal seams with depth may be directly related to the geological conditions. The brachyanticlinal structures of the Aralichevo deposit, as well as the oil domes are favourable containers for an accumulation of large amounts of gas (methane). This has been well demonstrated by the test-holes sunk in the Gorbunikha and Kopylikha anticlines.

Nearly all of the holes sunk in the crest of the anticlines produce large quantities of gas upon exposing the coal seams. Thus, for instance, hole No. 237, sunk in the crest part of the fold, upon the exposure of the highly disturbed rocks enclosing seam V (or V bis), produced up to 2200 m³ of gas per 24 hours, as shown by the measurements made by the present author.

All this points to vast reserves of gas confined to the crest of anticlines (the so-called gas domes are termed by the author «gas caps»). Particularly great reserves of methane are confined to lit-par-lit displacements of the zones of tectonic disturbances. When drilling hole No. 5 along the line of the main cross-cut of the Ordjonikidze shaft, during the drilling of seam IV, an intense production of gas was recorded, due to which the gas was throwing out of the hole the flushing water to a height of 1.5 m. Such an accumulation of gas; causing the presence of a large amount of gas in the shafts, makes us regard the combustible gas (methane) not only as an injurious factor, but also as a useful object for exploitation. When mining gas here from some dome-shaped structures of the coal deposits by means of drill-holes, two objects may be attained: degasing the coal seams, preparing them for working by shafts, and supplying the industry with fuel gas which is so necessary at the time of the National War.

No final conclusions as to the possibility of mining fuel gas in the Kuznetsk basin can be reached until supplementary reconnoitring and systematic hydrogeological and gas-chemical analyses and stationary observations are made; these will permit of a provisionary evaluation of the underground reserves of the gas and of its pressure with different modes of occurrence.

Institute of Geological Sciences,
Academy of Sciences of the USSR.

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MINERALOGY

MINERALOGY AND GEOCHEMISTRY OF THE LEAD-ORE-BEARING DEPOSIT MIRGALIMSAY IN SOUTH KAZAKHSTAN

By E. V. SHEVCHENKO and V. K. MONICH

(Communicated by V. G. Chlopin, Member of the Academy, 15. III. 1943)

The lead-ore deposit Mirgalimsay, situated in the Djambul district of Kazakhstan, contains fine-grained disseminated ores bedded in limestones of the Upper Devonian. The main ore deposit is of the character of a layer and is dated from the middle part of the Upper Devonian series, from the horizon of «middle banded» limestones, converted into banded dolomite rocks. Genetically this deposit is a telethermal type and is considered to be connected with the Variscian folding that form the regional structure of Karatau.

Ore hydrothermal metasomatism is chiefly concentrated in banded limestones with a special pseudo-oolitic microstructure of seaweed or colloidal origin. Terrigenous components which make up 2—7% of the total mass of the rock are represented by kaolinized minerals, feldspar, muscovite, sericite, biotite and separate grains of zircon, rutile, sphene, garnet, apatite, epidote, tourmaline, hornblende, magnetite, haematite.

At the beginning of the ore-process limestones are subjected to partial and irregularly distributed recrystallization of spotted and banded type. This was probably due to relatively cold carbonic acid thermae which transformed calcite into readily soluble dicarbonate of calcium. The irregular recrystallization of limestones increased the porosity of rocks. The subsequent dolomitization of limestone led, first, to the formation of disseminated grains of dolomite in the mass of limestone, and later on, of solid grained dolomites, attracted to the zones of ore mineralization. The main mass of dolomites bears a clearly metasomatic character, its deposition being accompanied by a very energetic corrosion of calcite and by the formation instead of it of rhomboid grains of dolomite. The concentration of carbon dioxide could not considerably increase at that period, because sulphates play a prevalent part in solutions towards the end of the period of dolomitization. It should therefore be assumed that the increased corrosive nature of the solutions is largely due to the rise of temperature.

After dolomite and partly along with it large ore amounts of barite precipitate which compose on the average about 38% of the ore mass. In a number of cases the two minerals are closely connected with each other spatially; the sequence of their separation is indistinct and variable, pointing to their being deposited at the same time. We are thus brought to the conclusion that the process of dolomitization came to an end under the influence of sulphatic magnesian-barite thermae on limestones.

From this moment the character of ore-bearing solutions is entirely changed and becomes more complicated. Sulphatic thermae give place again to carbonic acid thermae. In the ores of barite-dolomite rocks and in the zones which are oriented along layers, are deposited fine crystalline grains of complex carbonates of the type of ankerite, middle-grained dolomite, witherite and calcite. Ankerite

is represented by two varieties: one variety with a small content of iron (N_m = = 1.698—1.700); and another variety which is rich in iron and manganese (N_m about 1.760). In the course of oxidation of the ores these carbonates give typical coffee-brown ochres enriched in manganese compounds. Carbonate of barium; witherite with N_g somewhat less than 1.682 and N_p close to 1.530 also enters in the composition of the rocks.

The precipitation of sulphides begins without a sharp tectonic exposure of the mineralized zones. Hydrogen sulphide thermae enriched in metals became gradually admixed to carbonate thermae and at last entirely replaced them. The predominant sulphide is fine-crystalline galenite replacing dolomite, ankerite, barite and calcite. The content of galenite in ore stones is characterized by a considerable constancy. Sphalerite is represented by a dark-brown ferriferous variety and is a later ore mineral of the deposit. Its content in the ore varies from 0.68 to 2.2% (by weight). Both sulphides precipitated at the same time, but the period of the formation of sphalerite was shorter, owing to which partial adjustment of galenite to the grains of sphalerite is observable. Copper sulphides: chalcopyrite and tetrahedrite are found in small quantities. The general content of copper in ore is expressed by tenths and hundredths of per cent.

As to pyrite, this mineral is rather widely distributed in the deposit, composing about 1—2% of the main mass of ore. Its formation preceded that of the rest of sulphides; and it shows an independent development occurring at a great distance from the ore zones. Along with the sulphides some quantity of silver and a small quantity of gold are found. The mineralogical expression of these minerals in the ores of the deposit has not yet been determined. Data of analyses show that their content does not depend on that of galenite and sphalerite. Tetrahedrite can neither be the chief concentrate of these minerals because of its rare occurrence in the ores of the deposit. Data of spectrum analyses given in table below show that silver is met as admixture in a number of minerals of the deposit.

A feature characteristic of the second stage of mineralization is the localization of ore in thin short fissures. The fissure veins are 0.5—9 cm thick and 1—5 m long; their distribution is from 2 to 3 per m^2. The veins are composed by the same minerals as the ore bodies they cross. The only difference is the relative quantity of the minerals: the ore minerals play quite a subordinate rôle in the veins; among non-metal minerals calcite being evidently prevalent. Among ore minerals copper sulphides are widely distributed, while sphalerite is poor in iron and belongs to the cleiophane type. The characteristic feature of the process of vein mineralization is the clearly expressed tendency to the isolation of non-metal minerals, owing to which veins of nearly pure mineral matter of calcite, barite; witherite composition are prevalent.

A rather considerable distribution of vein mineralization not only in ores, but also at a distance from the ore zones points to the tectonic nature of the fissures and to the wide scale of renewed hydrothermal activity. The composition of the ore-bearing solutions becomes more complicated as compared with that at the end of the first phase; the ore-bearing solutions have apparently been renewed, their temperature became somewhat higher, because a certain quantity of sulphate thermae was added to the sulphide carbonate thermae. At the beginning of the second stage of mineralization along the system of small fissures circulated solutions which by their composition and even by the elements-admixtures (see the table) resemble primary ore-bearing solutions. The activity and the corrosive nature of these solutions were rather low; and the metasomatic processes were not developed. There are manifestations of a later oreless mineralization in the form of veinlets of calcite and aragonite; crossing and displacing the system of calcite veins of the second stage of mineralization.

Late minerals of lead are represented by cerussite and by a subordinate quantity of anglesite. In the upper zone of ochreous ores (at a depth of 3.5 m) these minerals form the whole mass of lead; but the enrichment of the ores with metal is not great. In ores opened by underground mining these minerals contain 60—70% of lead; about 50% falling to the share of cerussite. The usual sequence of the mi-

neral formation is distinctly observable—the inner rim of galenite is composed of anglesite; the outer larger rim is composed of cerussite. In sulphide ores of deep horizons there is only a thin broken rim or a thin layer of cerussite on galenite. Here also the portion of cerussite is about 20% of the total amount of lead. The portion of anglesite is nevertheless about 1%. In this case the phenomenon of direct oxidation of galenite does not play a great rôle.

The paragenetic complexes of the zone hypergenesis in Mirgalimsay are characterized by the presence of products of various geochemical processes. The most typical of them are the following: a) oxidation and hydratation (ochres, iron and manganese, gypsum, anglesite); b) leaching (enrichment of ochres with kaolinite and combinations of iron-manganese, dissolution of dolomite); c) sulphate-carbonate exchange (cerussite, smithsonite, gypsum, malachite, azurite); d) sulphide-sulphatic exchange (chalcosite); e) hypergenesis of the xerothermic climatic phase under participation of swampy waters (atacamite).

Additional features of the mineralization process are seen from the data on the distribution of minor elements in minerals of Mirgalimsay (see the table).

Distribution of Minor Elements in Minerals of Mirgalimsay
(after spectrum analysis data)

Elements	First stage						Second stage				Third stage		Hypergenetic minerals			
	Primary calcite	Barite	Pyrite	Ankerite	Sphalerite	Galenite	Chalcopyrite	Cleiophane	Galenite	Calcite	Calcite	Aragonite	Chalcosine	Yellow ochre	Coffee-brown ochre	Red-brown ochre
Silver	—	4	4	—	4	3	4	4	3	—	—	—	4	3	3	3
Lead	3	3	4	1	5	+	4	2	+	4	1	3	3	2	3	6
Zinc	3	—	2	1	+	3	2	+	3	—	—	1	1	2	2	—
Copper	3	3	3	1	4	3	+	3	3	4	2	2	+	2	2	1
Cadmium	—	—	—	—	3	—	—	3	—	—	—	—	—	—	—	—
Gallium	3	—	—	1	3	—	—	2	—	—	—	—	—	1	1	—
Germanium	—	—	—	—	3	—	3	—	—	—	—	—	+	—	—	—
Molybdenum	3	1	1	1	2	—	2	1	—	—	—	1	2	—	1	—
Iron	4	6	+	+	6	2	+	5	4	2	5	2	5	5	5	2
Manganese	4	4	2	+	4	3	3	2	4	2	2	2	3	4	4	3
Titanium	—	—	—	—	—	—	—	—	3	—	—	—	—	—	1	—

Note: 1) Spectrum analyses failed to reveal nickel, cobalt, tin, arsenic, antimony, bismuth, indium in any of the samples. 2) Symbols used:—(dash) element not found; + (plus) element enters the chemical formula of the mineral; 1—traces of spectral lines; 2—very slight lines; 3—slight lines; 4—stronger lines; 5—strong lines; 6—very strong lines.

Almost all minerals of the deposit including primary calcites are characterized by admixtures of non-ferrous metals (lead, zinc, copper). These metals were contained in the ore solutions nearly until the end of the process of mineralization. Silver disappears from the solutions by the end of the second stage. Molybdenum is characteristic for the first stage only. Cadmium, gallium and germanium have no wide distribution, being chiefly connected with sphalerites; germanium, with sphalerite and chalcopyrite.

Elements which are usually associated with the basal magma (nickel, cobalt) are not present in the minerals of the deposit. The absence of tin, bismuth, arsenic, characteristic of metallogenesis of Variscian polymetallic deposits of the districts adjoining Uzbekistan and Kirghizia (Kara-Mazar, Aktus), is noticeable.

Under hypergenic conditions there takes place concentration only of the most widespread elements; iron and manganese. Some small elements are leaching and migrating (germanium, cadmium, molybdenum).

Mining and Metallurgical Institute
of Kazakhstan.

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PLANT PHYSIOLOGY

TWO POTATO HARVESTS FROM THE SAME TUBERS

By T. S. TER-SAAKIAN

(Communicated by A. A. Richter, Member of the Academy, 5. IV. 1943)

It is known that the same tubers of potato can be used as seeding material only once during one season. Potatoes are planted in spring or summer and harvested in the middle of summer or in autumn, respectively.

I have found, however, that if potatoes are lifted in summer (June), the sound parent tubers can be planted for the second time, and new plants obtained from them. If lifted in autumn, the parent tubers usually show signs of rotting, or are lignified and affected with various diseases, and are, therefore, usually left in the field.

When the material of the foregoing year is planted in the spring or summer, the tubers of potato, especially if vernalized, develop nearly all their eyes in the soil. Tubers of any variety of potato have five to twelve eyes and more, according to the size of the tuber. From these eyes 4—6 tubers (more in the case of vernalization) are usually formed, and sometimes two buds appear from a single eye. It is obvious that the expenditure of nutrient substances for the development of the sprouts is very high in such tubers ([3],[5]).

It has been established ([1],[2],[4]) that in the southern regions of this Union, having a long growing season, two yields of potato are possible under industrial conditions. According to my experiments ([4]), the lifting of potatoes, after early spring planting, is to be made in the second half of July. For a second planting of potatoes in the same year (in summer) freshly harvested tubers may be used, which must be germinated and planted after grain crops into a preliminarily moistened soil. In a freshly harvested tuber I observed in most cases the formation of one or two sprouts from its apical young eyes. Under normal conditions of planting a newly lifted germinated tuber, used as a parent tuber, develops roots and a strong branching stem with a large leaf area.

In the autumn, at the time of the second harvest, the state of the parent tubers gives reason to suppose that the utilization of the nutrient substances stored during the growing period of the plant is rather limited. The eyes of such tubers are nearly all in the dormant state, and if they are found to be sound when lifted in autumn, they may, as a rule, be used as seeding material next spring.

The possibility of using summer-planted parent tubers lifted in autumn was explored by the author in the springs of 1940 and 1941. The experiments were made on the experimental field of the Plant Genetics Department of this Institute. For the sake of comparison the following seeding material (variety Lorkh) was planted:

1) Tubers grown from spring-planted tubers of the foregoing year lifted in summer (summer harvested tubers).

2) Tubers from summer planting of freshly lifted tubers; harvested in the autumn of the foregoing year (autumn harvested tubers).

3) Parent tubers from summer planting of freshly lifted tubers; harvested in the autumn of the foregoing year (parent tubers of summer planting).

The experiments were run with watering and under similar soil conditions. Planting was made with a spade. The date of planting was April 1 in 1940, and March 18 in 1941. The time when the plants appeared above ground and the time of flowering is given in Table 1. It should be noted that the plants from parental tubers looked perfectly sound in the spring, and their further development was normal.

Table 1

Origin	Year of harvest	Year of experim.	Exper. plot in m^2	Number of tubers planted	Number of plants formed	Time of germination (75%)	Time of 1st flowering
Summer-harvested tubers	1939	1940	18	100	100	10.V	30.V
Do	1940	1941	18	100	100	10.V	19.V
Autumn-harvested tubers	1939	1940	18	100	100	7.V	30.V
Do	1940	1941	18	100	100	28.IV	25.V
Summer-planted parental tubers	1939	1940	18	100	100	11.V	30.V
Do	1940	1941	18	100	100	10.V	20.V

It may be seen from this table that tubers of different origin sent forth plants and formed flowers almost simultaneously. Yet, it may be remarked that in the case of autumn-harvested tubers the plants appeared above ground with a certain delay, and acceleration was observed only later.

The wilting of leaves was first observed in the plants grown from summer-planted parental tubers. In most of them the assimilation activity expired in the middle of July, whereas the plants of the autumn-harvested tubers had not yet wilted at the time of harvesting.

The tubers were lifted on July 17 in 1940, and on July 14 in 1941. The results were as follows (Table 2).

Table 2

Origin	Year of harvest	Year of experim.	Exper. plot in m^2	Number of harvested plants	Weight of tubers per plant in g	Yield of tubers in kg	Number of plants perished from mole-cricket
Summer-harvested tubers	1939	1940	18	95	89	8.5	5
Do	1940	1941	18	81	130	10.6	19
Autumn-harvested tubers	1939	1940	18	91	265	24.2	9
Do	1940	1941	18	89	289	25.8	11
Summer-planted parental tubers	1939	1940	18	86	186	16.0	14
Do	1940	1941	18	87	158	13.8	13

From this table it is seen that the plants from autumn-harvested tubers gave a higher yield than those from summer-harvested tubers. It should be noted; however, that the yield of plants from parental tubers is half as large again as that of the plants from summer-harvested tubers.

The experiments carried out in 1941 under industrial conditions in two collective farms—village Armavir (2 tons of potatoes were gathered from $^1/_4$ ha) and village Jafarabad ($3^1/_2$ tons from $^1/_2$ ha)—have shown in a very convincing manner that parental tubers lifted in autumn together with the yield from summer planting of freshly harvested tubers, may be used for the second time as additional seeding material for spring planting.

Biological Institute.
Armenian Branch
of the Academy of Sciences of the USSR.

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PHYSIOLOGY

ACTION CURRENTS OF PAIN IMPULSES IN MAN, NORMAL AND AFFECTED BY CAUSALGIA

By V. S. RUSSINOV

(Communicated by N. N. Burdenko, Member of the Academy, 1. VI. 1943)

In 1939 the present author worked out in collaboration with S. A. Chugunov a method for recording the action currents of nerve trunk in man when electrodes are applied to the skin at a point where the nerve trunk is passing directly under the skin. Applying electrodes to *sulcus bicipit. intern.* at the points of passage of *n. medianus* and *n. ulnaris*, we were able to record the action currents of the nerve impulses transmitted along those trunks. The action currents were recorded by means of a sufficiently sensitive amplifier. Previously to applying the electrodes, the spot was determined precisely by irritating the corresponding nerve.

The rhythm of action currents as recorded for an arm at rest on an electrogram of *sulcus bicip. intern.* is 70—90 per sec. This rhythm should evidently be put down largely to the afferent impulses that continuously arrive from the periphery to the central nervous system.

With adequate stimulations of the innervation zone of the corresponding nerve, the electrogram of *sulcus bicip. intern.* shows definite variations in the rhythm and amplitude of the action currents. It should be noted that the rhythm varies according to the character of stimulation. Tactile stimuli applied to the palm surface of the index give upon the electrogram of *n. medianus*, taken from *sulcus bic. intern.*; a rhythm amounting on the average to 200 per sec. The stimulation of proprioceptive apparatuses by suspending load to the index gives upon the electrogram of the same nerve a rhythm varying with the load applied. The action currents of proprioceptive impulses differ from those of tactile impulses not only by their rhythm and amplitude, but by their shape as well.

Thus, if the integrity of the conductive function of a nerve may be evaluated after the general variation of the rhythm of an electrogram of a nerve trunk, after the shape of action currents one is able to identify the afferent system operative at the moment being.

The purpose of our subsequent work was to investigate the action currents of a nerve trunk of man in the case of pain stimuli, which has not yet been done.

The method was the same as used when investigating action currents of tactile and proprioceptive impulses. We recorded the action currents of pain impulses sent out from *n. medianus*. The lead-off electrodes were as usual applied to *sulcus bicip. intern.* to the skin, along the course of the nerve trunk. The pain stimuli were inflicted by pricking with a pin the palm surface of the index. When conveyed along *n. medianus*, the action currents of the pain impulses were led

off with the aid of a 4-cascade amplifier to an oscillograph, and recorded upon photo-paper. The test-object was placed in a screened room.

Fig. 1 reproduces a picture typical of the action currents of nerve in a normal man subjected to pain stimuli. According to their rhythm, amplitude and general shape, these currents differ sharply from the action currents of tactile and proprioceptive impulses. As may be seen on the electrogram, the action currents of pain impulses in a normal man consist of two parts: of a high-voltage spike and

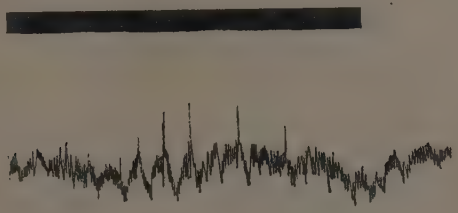

Fig. 1. Patient G., 19. III. 1942. Wounded 20. VIII. 1941. Electrogram from *sulcus bicip. intern.* Action currents of pain impulses.

of a lengthy potential whose duration is up to 0.2—0.3 sec. The rate of these currents is low, ranging from 2 to 7 per sec. Their rhythm is consequently much slower than that of action currents of tactile or proprioceptive impulses. Our data coincide with the results obtained by Adrian, Matthews and others who worked with nociceptive stimuli in animals.

Of great interest are the action currents of pain impulses in patients affected with causalgia, which is distinguished by its peculiar character of pain. Their analysis may be helpful in the approach to the question of the pathogeny of this disease.

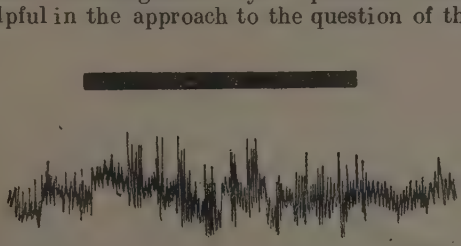

Fig. 2. Patient Z., 14. III. 1942. Wounded 8. VIII. 1941. Causalgia. Electrogram from *sulcus bicip intern.* Pain stimulus.

We made records of action currents of pain impulses for a number of patients displaying phenomena of causalgia. As an example, let us dwell upon patient Z. suffering from traumatic neuritis of *n. medianus*, *ulnaris* and *radius* as a result of a fire-arm wound with a fracture of the humerus. The sensitivity in the zone of innervation of *n. radialis* was almost completely restored within 5 months; by the end of this period it was the disordered sensation and paresis of muscles innervated by *n. radialis* and especially by *n. ulnaris* that came to the foreground. The slightest tactile stimulation not only aggravated the pain, but also aroused motor reaction. The electrogram revealed a strong electric activity due largely to the action currents of muscles, which impeded the examination considerably. We succeeded in recording the action currents of pain impulses in the patient not until the phenomena of causalgia began somewhat to attenuate.

Tactile stimulation of the palm surface of the index and of the little finger provoked on electrograms of *sulcus bicip. intern.* separate spikes pointing to the fact that the sensory conductors of *n. medianus* and *n. ulnaris* were not entirely broken. The integrity of motor fibres was evidenced by the presence of a reaction which ensued as soon as those nerves were tested for electro-excitability.

Pain stimuli applied at this period to the index produced in the electrogram changes different from those that are usually observed in response to similar stimuli in normal test-objects.

When a patient suffering from causalgia is subjected to a pain stimulus; the electrogram shows potentials of a lengthy order, with a group of spikes on each wave of the prolonged potential (Fig. 2). These group «explosions» against the background of a slow potential form one of the typical features of the electrophysiological activity of the nerve trunk in the case of causalgia. It seems probable that if, unlike our case, the causalgia phenomena are sharply pronounced, these group discharges have spikes of a considerably wider amplitude.

What is the interpretation to be given to the group «outbursts» of electrical activity observed in the case of causalgia? We recorded action currents of *n. medianus* above the spot of affection; consequently, the afferent impulses will have passed the affected section of the nerve before the current of action will be recorded. The «outburst» reaction which proceeds according to the principle of «single tetanized contractions» is a type of reaction well-known in the physiological literature dealing with the efferent peripheral system. It has been shown by Wwedensky that in case a focus of local stationary excitation is created by means of subthreshold stimulation in the motor nerve somewhere on the way of a single impulse, such a focus may respond to a single impulse out of a whole series of impulses. The Wwedensky reaction was investigated in detail by Samoilov and Kisselev who made use of the electrophysiological method. A similar type of reaction of the peripheral nerve is observed in case a single impulse passes a section of a nerve whose functional state has been changed by an anode of d. c.

The «outburst» nature of the excitation, which has been observed by us in the peripheral nerve trunk for afferent impulses in the case of causalgia, seems to be due to a peculiar corroboration between the transient impulse and the local excitation of the affected section of the nerve itself.

The phenomenon that is faced by us in the case of causalgia is certainly more complicated than the «outburst» type of reaction observed in the motor peripheral system; the physiological mechanism of the reaction seems, however, to be essentially alike. It may also be that the skin receptors react in a different way in the normal state and in the case of causalgia; but this question, as well as many other questions related to causalgia, are beyond the scope of the present investigation.

Institute of Neurosurgery.
Moscow Brain Institute.

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MORPHOLOGY

ON POST-EMBRYONIC TRANSFORMATIONS OF *FOR. OVALE* OF THE HEART IN HIGHER MAMMALS AND MAN

By W. N. SHEDENOW

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Our previous work was concerned with the ultimate morphogenetic processes in the region of *foramen ovale* of the foetal heart and with the functional structure of its valve ([1],[2]). No less interesting from the point of view of biomorphology is the mechanism of the closure of *for. ovale*, and the post-embryonic transformations it undergoes.

The material and method used in this work have been similar to those used in our previous investigations.

As found by us, the mechanism of the closure of *for. ovale*, which takes place in higher mammals and man upon birth, is a very complicated process. It may be reduced to three factors: a) falling together of the valve, whereby *for. ovale* becomes functionally shut by it; b) rearrangement of its structure and c) endocardial coalescence of the valve with the adjoining walls of the auricle. As a result, *for. ovale* together with the valve undergo involution, while the valve proper becomes reduced. The falling together of the valve is known to occur as a result of the fact that pressure becomes lower in the right and higher in the left auricle*. The character of the process depends on the shape of the valve and is not the same in different animals.

Immediately after the valve fell together, its structure undergoes rearrangement as a result of functional inactivity. The reconstruction largely consists in a gradually developing process of atrophy of the muscular tissue, and its replacement by connective tissue (cicatrization).

The reconstruction process of the valve usually involves after a certain period endocardial coalescence of the adjoining regions of the valve and the *limbus*, which overlap each other (*septum atriorum*, medial wall of the *auriculum*).

Worobiev ([6]) was inclined to interpret the preliminary coalescence of endocardial foils as a result of their irritation by the blood flow. This cannot be assumed unless we were to admit the existence of a preliminary rearrangement of the inner structure of the valve itself. It is this active process of reconstruction of the valve tissue that can alone, to our opinion, involve the endocardial tissues into the active state necessary for the coalescence process followed by concretion. And this

* Barclay and Franklin have shown ([3]) that *for. ovale* is not closed by the valve directly upon birth, it is not shut at once, as it is commonly believed, but undergoes gradual coalescence under the action of its elastic tissue, because of the change of pressure in the auricles.

is the more probable as the endocardium of the *limbus* owes its origin to the fusion with the adjoining inner wall of the valve proper (*septum* 1) (1).

In the absence of thread-like fibres, the coalescence of the free end of the valve starts at its edges which are grown to the *limbus*. On account of the constriction of the valve itself, this gradually leads to the obliteration of its outlet or channel. The obliteration usually takes place not at the very edge of the valve, but at some distance inward. The end of the valve may therefore remain open even after a complete coalescence of the duct, in the form of a semi-lunar fold on the left part of the partition between the auricles.

The basal part of the valve which is directly facing *for. ovale* is the last to undergo reconstruction. The constriction of this part leads to a tight closure of *for. ovale*.

The thick muscular layer of the valve atrophies very slowly. It is usually preserved by the animal in a deformed shape throughout its life.

In a number of cases the channel (duct) does not close at all in any of its sections; it remains then all along the life of the individual at a certain stage of obliteration. Such cases should not be confounded with anomalies in the development of *septa* I and II, that are not infrequently observed in this part of the heart.

Schemes of the stages of closure of *for. ovale* in man have been suggested by more than one author. The most successful of them seems to us those by Seki and Minkin and Svetlova (2,3). To our opinion, the following modes of this process should be distinguished both in man and animals: 1) complete coalescence of *for. ovale*, without any traces of the formed duct; 2) coalescence of *for. ovale*, a bay being left behind; 3) *for. ovale* functionally closed, but not grown together (a small constricted duct is preserved); 4) open *for. ovale* (free duct preserved, alongside of depression of the valve); 5) *for. ovale* gaping, in the case of underdeveloped or totally lacking valve.

Cases of *for. ovale* persistent throughout the life, as well as other phenomena in adult animals preserving embryonic characters of the heart, should be regarded as phenomena of embryonalism. These phenomena are of most frequent occurrence in cattle. It should be noted that in wild species of ruminants the closure of *for. ovale* with the valve proceeds in a much more perfect form than in the corresponding domesticated species*. This makes us believe that the breakdown of the correlative dependence (disintegration, according to Schmalhausen) between the reduction of the valve and the process of coalescence of the endocardial foils is associated with a sharp abatement of natural selection as a result of domestication.

Taking into account that in the left auricle the pressure is higher than in the rigth one, it should theoretically be assumed that in a normal state (valve present) a considerable mixture of blood through the existing communication is scarcely possible during the systole of the auricles, especially in animals. This is more likely to be promoted by their diastole**.

Anatomical Institute
of the Veterinarian High-School.
Chkalov.

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* According to data obtained at the Laboratory for evolutionary morphology of domestic animals of the Institute of Evolutionary Morphology, Academy of Sciences of the USSR, and at the laboratory of Prof. S. N. Bogoliubsky.

** Intense work (especially during pregnancy), or pathological condition of the heart may evidently bring about the mixture of blood and the harmful consequences that may take place.

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PALAEONTOLOGY

YOGLINIA n. g., LATEST PTERASPID FROM THE MIDDLE DEVONIAN OF THE LENINGRAD DISTRICT

By Dm. OBRUCHEV

(Communicated by A. A. Borissiak, Member of the Academy, 11. V. 1943)

The order *Pteraspiformes* includes the best known genus of the subclass *Heterostraci—Pteraspis*. This order by the time being consists of a single family, *Pteraspidae*, occurring in the Uppermost Silurian (Downtonian) and in the Lower Devonian. It is generally agreed that this order has become extinct towards the end of the Lower Devonian. There are, however, certain indications that the age of some strata containing pteraspid remains may be younger than Lower Devonian. Thus, Wl. Zych (1, p. 63) referred the upper part of the Podolian Old Red to the Middle Devonian. A. Heintz informed me that the Wood-Bay series of Spitzbergen bears a fish-fauna, including along with Lower Devonian forms (acanthaspids, pteraspids, cephalaspids) also Middle Devonian genera *Homostius*, *Heterostius*, *Actinolepis*. We may therefore assume that the pteraspids still lived in the Middle Devonian. In this connexion gain in interest the remains of a new and original pteraspiform fish, found by the author in 1931, in the upper part of Middle Devonian of the Yoglina river.

Not only a new genus, but even a new family are to be founded for the form under description. *Yoglinidae* differ from typical *Pteraspidae* in having: 1) a thick cellular layer of psammosteid type instead of a layer of hexagonal cancellae; and 2) dentine ridges coarser than in pteraspids, although finer than in psammosteids.

The genus *Yoglinia* differs in its broad pointed cornuals, developed stronger than in any other pteraspid and serving as supporting organs, which is obvious from their outer margin being worn off and covered with massive spongiosa. The only species is:

Yoglinia bergi n. sp.

Holotype: cornual plate (see figure), Palaeontological Institute, Moscow, No. 220/504.

Locality: river Yoglina, tributary of Zhelcha, Gdov region; Leningrad district; near Teppo's watermill.

Age: Middle Devonian (Givetian), uppermost Luga beds.

Description. The holotype is a broad plate, thin at the base, tapering to the distal end. The outer margin is direct, thickened, the hind margin, thin. The anterior end is broken off.

On the convex dorsal surface there is a 3 cm broad zone along the inner margin,

devoid of dentine ridges. On the remaining surface, the dentine ridges are essentially longitudinal, radiating somewhat from the anterior end to the hind margin. The ridges are rather short, often interrupting, the next ridge beginning opposite the space between two preceding ones. They are asymmetrical, their proximal slope being gentler, with more prominent wrinkles and denticles. There are 30—40 ridges to 1 cm (across their breadth).

Left cornual plate of *Yoglinia bergi* n. sp. (holotype). Dorsal side. Drawn by N. A. Yanshinov, ×1

On the concave ventral surface the ridgeless zone is broader, about 3.5 cm. The ridges run in the same manner as on the dorsal side. The anterior-lateral part of the base is hollow, the thin walls of this hollow are broken off, especially ventrally.

Besides the holotype, there is only one fragment at hand.

C o m p a r i s o n. The described plate is much similar in form to the branchials of *Psammosteus*, but not homologous with them. This is clear from its convex dorsal surface being an outer one, i. e. its ridgeless base having been covered with skin; the concave ventral surface, however, is the inner one, because its base was covered with soft parts of the body, penetrating also into the hollow of the plate. Hence it follows that the plate pertained to the dorsal surface of the body, and covered the muscles and branchial pouches of the animal from above, and not from below, as do the branchial plates in psammosteids. Thus, it is homologous with the cornuals of psammosteids and pteraspids.

In the psammosteids the cornuals, known as yet only in *Drepanaspis gemündenensis*, are small, and function only as covers bordering the medial margin of the branchial opening. The branchials, however, in the course of the evolution

are transformed from the covers first into gliding planes (when swimming), and afterwards into supports (upon the bottom). In the pteraspids, on the contrary, the same course of evolution is followed by the cornuals, while the branchials continue to cover the branchial pouches and ducts.

We have here a case of divergent evolution (since psammosteids descend from early pteraspids), proceeding in both branches by way of parallel changes, but in analogous elements instead of homologous ones.

Yoglinia is a late, as yet the latest, member of the pteraspid stock. This stock passed, like the psammosteids, from a marine-lagoonal life to a life in running water, in rivers. As in the psammosteids, there arose a need in strong supports in moving on the bottom against the current. Analogous changes in the environment called forth analogous changes in the organization, but these changes involved different elements of the external skeleton. The cause of this lies in the hind lateral angles of the trunk being formed, in psammosteids (*Drepanaspis*) by branchials, in pteraspids, by cornuals. And it is self-evident that the gliding planes or the supports would evolve just from these angles.

Similar parallel series of changes in the *Carnivora*—the development of the sectorial teeth from different pairs of cheek-teeth—were interpreted by Abel ([2], p. 282—284) in favour of his «biologisches Trägheitsgesetz». However, there is a possibility of a simpler solution of this problem, without any recourse to orthogenesis. Adaptations, even not «ideal» ones, are nevertheless useful for the organism and subject to selection, notwithstanding their ultimate relative unsuccessfulness, leading to the extinction of such branches when they come across more successfully adapted ones.

Palaeontological Institute.
Academy of Sciences of the USSR.

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